A Preliminary Implementation Of Joint Source And Channel Coding In MIMO System

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Abstract – The coding process in MIMO system is directly related to the effort of maximizing channel gain, namely diversity gain and multiplexing gain. In contrast to the channel coding extensively investigated in various research-literatures on MIMO system, the source-coding method for the system is typically not expressly covered. The justification for this common approach is the Shannon Separation Theory, which states that source and channel coding can be separately done without affecting the system performance. More recent researches show that the Shannon Separation Theory does not hold for certain conditions, which encourage researches on joint source and channel coding methods.

In this paper a preliminary joint source and channel coding in the form of Rate Compatible Punctured Convolutional Code (RCPC Code) applied as an Unequal Error Protection (UEP) to the source stream is proposed. The RCPC code is achieved by puncturing a low rate 1/N code periodically with period P to obtain a family of codes with rate P/(P + ℓ) where ℓ can be varied between 1 and (N-1)/P. Simulations are done to examine the performance of the code. Results show that the proposed RCPC code enables unequal error protection to the source information ordered with increasing importance. A further research is planned to implement RCPC on MIMO system.

Keywords – MIMO, Rate-Compatible Convolutional Code, Unequal Error Protection

I. INTRODUCTION

Current researches propose Multiple-Input Multiple-Output (MIMO) as the main technology to support digital wireless communications systems of the third generation and beyond. MIMO is an extension of smart antenna technology, in which both the transmitter and receiver sides of the system are equipped with multiple antennas. The technology is especially appealing due to its potential of linear-growing capacity as a function of antenna numbers, and its capability to exploit multipath fading which is a pitfall in conventional wireless communication system.

The coding process in MIMO system is directly related to the effort of maximizing channel gain, namely diversity gain and multiplexing gain. The diversity gain characterizes the system’s robustness against errors, while multiplexing gain characterizes the system capacity. One of the theoretically suitable channel coding method for MIMO is Space-Time Code [1], which later extends into Space Time Trellis Code [2] and Space Time Block Code [3].

In contrast to the channel coding extensively investigated in various research-literatures on MIMO system, the source-coding method for the system is typically not expressly covered. The justification for this common approach is the Shannon Separation Theory, which states that source and channel coding can be separately done without affecting the system performance [4]. More recent researches show that the Shannon Separation Theory does not hold for certain conditions [5], which encourage still more researches on joint source and channel coding methods.

A recent approach of a joint source and channel coding method in MIMO is investigated in [6] mainly to obtain a performance parameter named expected distortion, while not specifying the code rate of the system.

One of the known concepts of joint source and channel coding is Unequal Error Protection (UEP) in which source information is given different level of protection according to its importance. A Rate-compatible Convolutional Code (RCPC) is a method which enables a system to have different code rates to match the source information requirements, and therefore can be applied in UEP. It is of our interest to examine the possibility of implementing RCPC codes on MIMO system.

The second section of the paper will cover the definition of RCPC code and its application to UEP is considered. The third section gives preliminary experimental result, and conclusion is given in the last section.
A family of RCPC codes is derived from a mother code of rate \( R = 1/N \) and memory \( M \) with generator tap matrix \([7]\)
\[
g = N \begin{bmatrix} g_1 & \cdots & g_M \end{bmatrix}
\] with tap connection \((g_{ik}) \in (0,1)\), where 1 denotes a connection from the \( k \)-th shift register state \( k \cdot \ell \) to the \( i \)-th output. Puncturing period \( P \) and \( N \) determine the code rate \([7]\)
\[
R = P/(P+\ell) \quad \ell = 1,\ldots,(N-1)P
\] from \( P/(P+1) \) to \( 1/N \). RCPC codes are punctured codes from a mother code with puncturing matrices \([7]\)
\[
a(\ell) = N \begin{bmatrix} a_{\ell}^{(i)} \end{bmatrix}
\] with \( a_{\ell}(l) \in (0,1) \) and 0 denotes puncturing. Puncturing decoder whilst using codes with different rates, instead of switching between an array of encoders and decoders. The puncturing of codes will provide a rate-compatibility with the following rules \([7]\)
\[
\text{if } a_{\ell}(\ell_0) = 1 \text{ then } a_{\ell}(\ell) = 1 \text{ for all } \ell \geq \ell_0 \geq 1 \quad (4a)
\]
or equivalently
\[
\text{if } a_{\ell}(\ell_0) = 0 \text{ then } a_{\ell}(\ell) = 0 \text{ for all } \ell \leq \ell_0 \leq 1 \quad (4b)
\]
As RCPC codes are in the class of convolutional codes requiring Viterbi decoding in the receiver, the optimality criterion used follows Viterbi’s upperbound error event probability \([8]\)
\[
P_e \leq \frac{1}{P} \sum_{d=0}^{\infty} \bar{a}_d P_d \quad (5)
\]
and error probability
\[
P_s \leq \frac{1}{P} \sum_{d=0}^{\infty} c_d P_d \quad (6)
\]
with \( P_d \) is the probability that the wrong path at distance \( d \) is selected. The distance spectra \( \bar{a}_d \) and \( c_d \) should be as small as possible and depends on \( N, M, P, g, \) and \( a(\ell) \). The transmission scheme for RCPC code over a nonfrequency-selective fading channel with multiplicative distortion \( a_F \) is shown in Fig. 1. The expectation value \( E(a_F^2) = 1 \) and \([7]\)
\[
p_{aF}(a_F) = 2 a_F (1+\frac{C}{M}) \exp \left[ -a_F^2 \left(1+\frac{C}{M}\right) \right] \cdot I_0 \left( 2 a_F \sqrt{\frac{C}{M}(1+\frac{C}{M})} \right) \quad (7)
\]
For Ricean channel, \( C/M \) is the ratio of the direct to the diffusely reflected signal energy, while for \( C/M = 0 \) and \( C/M = \infty \) a Rayleigh fading and a Gaussian channel is obtained, respectively. In the fading case a perfect interleaving is assumed, implying that the Channel State Information (CSI) values \( a_F \) and the received code values \( y \) are statistically independent with density function \((7)\). The channel SNR is measured by \( E_s/N_0 \) where \( E_s = E_d/R \) due to the varying rate of the code.

The Viterbi algorithm for decoding RCPC code uses the metric \([7]\)
\[
\lambda_j = \sum_{i=1}^{\infty} a_i a_{\ell} x_i^{(j)} y_j \quad (8)
\]
where \( x_{ij} \) denotes the transmitted binary information symbols, \( a_{\ell} \) denotes the fading factor with density \( p(a_F) \), and \( y_j \) denotes the received symbols.

The system performance in Rayleigh fading environment by soft decision on \( y_j \) using the full CSI with perfect estimation is \([7]\)
\[
P_s \leq \frac{1}{2 \left( 1 + E_s/N_0 \right)^{\frac{1}{2}}} \quad (9)
\]
The required BER after decoding a group of information bits is one example of SSI. Assuming that in a block of $n$ information bits there are $K$ groups with $n_k$ information bits in the $k$th group requiring a BER of $P_{bk}$ after decoding. $P_{bk}$ would be the SSI and $[7]$

$$\sum_{k=1}^{K} n_k = n$$  \hspace{1cm} (10)

Similar to [7], an example where source bits are ordered according to their relative importance and the required $P_{bk}$ is given in Table 1. It is our interest to use only one encoder and one decoder for this purpose, instead of separately encoding $K$ groups of information using $K$ different encoders and $K$ different decoders which might complicate the system.

<table>
<thead>
<tr>
<th>Code index $\mathcal{A}_k$</th>
<th>$R_k$</th>
<th>$a(\mathcal{A}_k)$</th>
<th>$d_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>$n_4$</td>
<td>$n_3$</td>
<td>$n_2$</td>
</tr>
<tr>
<td>&quot;0&quot;</td>
<td>$\mathcal{A}_1$</td>
<td>$\mathcal{A}_2$</td>
<td>$\mathcal{A}_3$</td>
</tr>
</tbody>
</table>

RCPC codes are well suited for this constraint. As shown in Table 1, the ordered information bits are shifted into the shift register of a $1/N$, $M$ stages RCPC coder.

The first group consisting of $n_1$ information bits is coded using $a(\mathcal{A}_1)$, and as the first bit of the second group enters the encoder, the system immediately switches to using puncturing table $a(\mathcal{A}_2)$. The system will switch to puncturing table $a(\mathcal{A}_3)$ as the first bit of the third group enters the decoder, and the process carries on until all information bits are transmitted. The process is terminated after the group $n_k$ by shifting $M$ "0" bits into the shift register. This implies transmitting $M/R_k$ overhead bits to terminate the trellis, and the average code rate is then [7]

$$R = \frac{\sum_{i=1}^{K} n_i}{\sum_{i=1}^{K} n_i - (P + l_1)/P + M(P + l_2)/P}$$  \hspace{1cm} (11)

III. IMPLEMENTATION OF RCPC CODES ON MIMO SYSTEM

To be able to implement joint source and channel coding to MIMO system, it is proposed to implement unequal error protection (UEP) to the system. The information bits are ordered according to their relative importance as implied in Table 1, and the more important bits are given stronger protection. As the design of an ideal UEP which adapts channel coding to a multimedia source bits is very complex, in this preliminary work the source bits are modeled as discrete time continuous amplitude source, namely

$$\{s_k\}_{k=1}^{\infty}, s_k \in \mathbb{R}$$  \hspace{1cm} (12)

$S$ different source information bits are transmitted using $M_t$ transmit antennas in a Rayleigh fading environment, and $M_r$ received antennas are employed in the receiver side. The transmission using multiple antennas result in a matrix channel. Fig. 3 shows the block diagram of a MIMO system.

![Diagram of a MIMO system](image)

Fig. 3 Diagram of a MIMO wireless system

Assuming the channel undergoes flat fading and information signal $s$ is transmitted by the $t$-th antenna, each of the receive antenna will see a complex-weighted version of the transmitted signal. A signal arriving at the $r$-th is denoted $h_{rs}$, where $h_{rs}$ stands for channel response between the $t$-th transmit antenna and the $r$-th receive antenna. The vector $[h_{10}, h_{20}, ..., h_{30}]$ is the channel response between the $r$-th transmit antenna and all receive antennas.

The channel matrix for the MIMO system ($H$) can be written in a matrix notation, that is
If signal vector \( \mathbf{x} = [x_1, x_2, \ldots, x_M]^T \) is transmitted consecutively by the 1st, 2nd, ..., up to the \( M \)-th transmit antenna, the arriving signals on the receive antennas can be written as:

\[
\mathbf{y} = \mathbf{Hx} + \mathbf{z}
\]

where \( \mathbf{z} \) denotes the channel noise. The proposed MIMO system with UEP is shown in Fig. 4a and 4b.

The source significant information (SSI) is as explained in the previous section, while the channel state information (CSI) contains fading depth information. The adaptive modulation is proposed as it is theoretically possible to be implemented alongside RCPC codes whose rate varies with the source information bits, but will not be covered in this paper.

The code rate is punctured from a mother rate of \( \frac{1}{2} \), using an encoder with memory \( M = 2 \) and generator matrix

\[
\mathbf{g} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\]

Fig. 5 shows the rate-compatible puncturing scheme used in the simulation, where \( P = 4 \), and the puncturing table used is

\[
a(1) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}
\]

implying a rate of 4/5.

Fig. 5 Encoding scheme using RCPC codes

After a period of \( P=4 \), the group of bits use another puncturing table namely

\[
a(2) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\]

which implies a rate of 4/6, and this rate can still be lowered further to 4/7, 4/8 using incremental redundancy. As the focus in this paper is the UEP using RCPC, interleaving, modulation as well as deinterleaving and demodulation are bypassed and the decoder performance is measured.

As the transmitted signal originate from multiple antennas, the rate is divided by the number of the transmit antenna \( M_n \) resulting in

\[
R = \frac{\sum_{i=1}^{K} n_i \cdot (P + l_i) / P + M(P + l_i) / P}{M},
\]

(14)

To ensure good performance, it has to be guaranteed that during the transitional phase between two matrices \( a \) (\( \xi \)) and \( a \) (\( \xi_{\alpha_1} \)), the distance properties of all paths originating in code \( \xi \) do not suffer a loss of distance. The rate compatibility rule (4a) ensures that the \( \xi_{\alpha_1} \) code does not puncture any “1” in the path, that is, maintaining a certain maximal rate. Simulation result in Fig. 6 shows that the BER can be kept low at the transition between rates, and for 128 information bits BER can improve significantly between rates 4/5 to \( \frac{1}{2} \).
IV. CONCLUSIONS

A joint source and channel coding in the form of unequal error protection is implemented in a MIMO system by using rate-compatibility punctured convolutional codes (RCPC codes). Simulation is done using code rate 4/5, 4/6, 4/7 from a mother code of 1/2, and the code performance is measured by without taking modulation and interleaving into account. A single user Rayleigh fading environment is assumed. Result shows that RCPC enables unequal error protection, in which more significant bits are given stronger protection by rate 1/2 to get BER of $10^{-4}$. Further investigation is planned to include modulation and multiplexing scheme in various conditions.

REFERENCES


