Introduction to Wavelet-Based Image Compression

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Outline
- Introduction to wavelet and wavelet transform
- Image compression scheme
- EZW – A wavelet-based image compression algorithm
- Overview of JPEG 2000
- Conclusion
- Reference

Wavelet and Wavelet Transform
- Wavelet
  - Small wave
- Wavelet transform
  - A way to decompose signal (just like Fourier transform)
  - A time-frequency analysis approach
  - Suitable for non-stationary signal
  - Notice gross features with large window
  - Notice small features with small window

Why Do We Use W.T.
- Fourier transform?
  - Only give what frequency components exist in the signal
  - No information about time
- STFT?
  - Unchanged window
  - Dilemma of resolution
    - Narrow window -> poor frequency resolution
    - Wide window -> poor time resolution
  - Uncertainty principle

Time-Frequency Resolution of W.T.

Comparison

From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10
Multi-Resolution Analysis (MRA)

- Wavelet transform
  - An alternative approach to the short time Fourier transform to overcome the resolution problem
  - Similar to STFT: signal is multiplied with a function
- Multi-resolution analysis
  - Analyze the signal at different frequencies with different resolutions
  - Good time resolution and poor frequency resolution at high frequencies
  - Good frequency resolution and poor time resolution at low frequencies
  - More suitable for short duration of higher frequency; and longer duration of lower frequency components

Multi-Resolution Analysis (MRA)

- Scaling function: \( \phi(x) \)
  Define: \( \phi_j(x) = 2^{-j/2} \phi(2^{-j} x - k) \)
  Therefore \( \psi_j(x) = \text{span} \{ \phi_j(x) \mid k \in \mathbb{Z} \} \)
- Wavelet function: \( \psi(x) \)
  Define: \( \psi_j(x) = 2^{-j/2} \phi(2^{-j} x - k) \)
  Therefore \( \psi_j(x) \subseteq \psi_{j+1}(x) \)

Multi-Resolution Analysis (MRA)

- Obtain \( V_j = \cap \psi^j \)
  Recursively:
  \( V_j = V_{j+1} \cap \psi_{j+1} \)

Any function \( f(x) \) can be expanded as

\[
f(x) = \sum_{j} c_j \phi_j(x) + \sum_{j} d_j \psi_j(x)
\]
Approximation Detail

Structure of Wavelet Transform

- Analysis is just "filtered" and "down-sampled"

Example of 2-D W.T.

- 2 levels of transform: 7 subbands
- 3 detail subbands
**General Image Compression Scheme**

- Transform
- Decorrelate spatial signal
- Quantization
- Drop information based on HVS
- Entropy coding
- Encode symbols into bit-stream

**Embedded Zerotree Wavelet (EZW) Coder**

- A quantization and coding strategy
- Incorporates characteristics of wavelet decomposition
- Outperform some generic approach
- Fundamental concept of other wavelet-based coder
- Can be decomposed into two parts:
  - Significant map coding using zerotree
  - Successive approximation quantization

**Significant Map Coding Using Zerotree**

- Scan order: From lower subband to higher subband

**Significant Map Coding Using Zerotree**

- Four types of Label
  1. Positive significant
  2. Negative significant
  3. Isolated zero
  4. Zero tree root

For each coefficient:
Give a label based on predefined threshold $T$

$$T = 2 \log_{2} \frac{T_{min}}{10}$$

**Successive Approximation Quantization**

- A refinement process
- Multi-pass scanning of coefficient using successive decreasing threshold
After this two step, we finish one iteration. 

\[ T_i = T_{i-1}/2 \] (reduce the threshold) 

Repeat until target fidelity or bit-rate is achieved.

Why Another Still Image Coding Standard?
- JPEG cannot fulfill the advanced requirements of today
  - Better quality and compression efficiency
  - New demands such as scalability and interoperability
  - New application area imposes some new requirements.

Features of JPEG2000 (1/2)
- Superior low bit-rate performance
  - Network image transmission
  - Continuous-tone and bi-level compression
  - Compound documents with images and text
  - Lossless and lossy compression
  - Medical images
  - Progressive transmission
  - Web browsing

Features of JPEG2000 (2/2)
- Region-of-interest (ROI) coding
- Open architecture
  - Allow to optimize the system
  - Robustness to bit errors
    - Transmission over wireless communication channel
  - Protective image security
    - Watermarking, encryption...etc

Example of Spatial Scalability
JPEG 2000 embedded bitstream (single layers)

- All images are decoded from the same bitstream.
Example of ROI

Subjective Quality (0.1bpp)

Subjective Quality

JPEG2000 Compression Engine

The whole compression engine can be decomposed into three parts:
- Preprocessing
- Core processing
- Bit-stream formation – Not included in this talk

Preprocessing

Image tiling

Table 1: The Effect of Tiling on Image Quality

<table>
<thead>
<tr>
<th>Tiling</th>
<th>No Tiling</th>
<th>tiles of size 128 x 128</th>
<th>tiles of size 64 x 64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bit Rate (b/yr)</td>
<td>0.125</td>
<td>24.75</td>
<td>25.42</td>
</tr>
<tr>
<td>0.25</td>
<td>26.09</td>
<td>26.09</td>
<td>23.98</td>
</tr>
<tr>
<td>0.5</td>
<td>28.27</td>
<td>27.79</td>
<td>26.80</td>
</tr>
</tbody>
</table>

PSNR (in dB) for the color image "old" (of size 720 x 480 pixels per component)
Preprocessing

- DC level shift
- Subtract each pixel value by 128 \(2^{8(p-1)}\)
- Component (Color) transformation
- Can be lossy or lossless

\[
\begin{array}{c|cccc}
T & 029 & 055 & 014 & 03 \\
C_x & 8.36675 & 0.53126 & 0.0314 & 0.0131 \\
C_y & 85 & -0.4109 & -0.60131 & -0.60131 \\
\end{array}
\]

Without/With Color Transform

Component (Color) transformation can be lossy or lossless.

Core Processing

- Wavelet transform
- Can be reversible (lossless) or irreversible (lossy) according to applications
- The standard use separable 1-D DWT for implementation

Core Processing

- Filter coefficient

| Table 4. 1D GOLF 5/3 Analysis and Synthesis Filter Coefficients. |
|-----------------|-----------------|-----------------|
| Analysis Filter Coefficients | Synthesis Filter Coefficients |
| Low-Pass Filter \(h_0\) | High-Pass Filter \(h_1\) | Low-Pass Filter \(g_0\) | High-Pass Filter \(g_1\) |
| 0 & 6.8 | 1.1 | 6.8 |
| 3 & 1.8 | -1.2 | 1.2 |
| 0.25 & 1.8 & -1.8 |

Core Processing

- Quantization
- Scalar quantization
- \[ q \left( u, r \right) = \text{sign}(u, r) \left\lfloor \frac{u, r}{\Delta} \right\rfloor \]
- Entropy coding
- EBCOT (Embedded Block Coding with Optimal Truncation)
- A kind of arithmetic code
- Descendant of EZW

Conclusion

- Wavelet analysis is powerful for application which we concern different extent of detail
- Image compression is one of the major applications utilizing wavelet transform
- EZW algorithm contains fundamental idea of other wavelet-based coder
- JPEG 2000 is a new standard providing a wide range of functionality utilizing wavelet transform, which is superior to other still image coding standard
Thank You!

Reference