Field Oriented Control of An Induction Motor Speed Sensorless with Current Vector Controller, direct-quadrature Current Compensator and Full Order Observer In direct-quadrature Axis

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Abstract The Observer is used to estimate three-phase induction motor speed sensorless, usually in alfa-beta axis, in this situation an extra transformation is needed to add compensator because the flux model is laid in dq-axis. In this paper, a new method to estimate the speed of induction motor drive with observer laid in dq-axis is proposed. The actual motor model used alfa-beta axis, but the observer use the motor models in rotor flux oriented control - RFOC, and also the different models of motor drives that be able to used between the actual and estimated one. The simulation results in C-MEX S-function Matlab/Simulink 6.5 show that full order observer gives better performance and in much better than reduced order observer.

Keyword : Current vector, induction motor, full order observer dq-axis and dq current compensator

I. INTRODUCTION

From the motor issues, that all the classical motors (DC, permanent magnet synchronous, and induction motors (IM)) have been put into operations. As well known, IM proceed great advantages over other kinds of motors, namely, simpler construction, smaller size, lower maintenance, better reliability, relatively cheaper, etc.[1]. The crucial factor in every motor drives is how to control the speed of the motor. The motor speed can be measured by the tachometer or another sensor, but this is not effective since it costs more expensive and many sensors not able to detect at very low and very high speed. Nowadays, when the knowledge of human being and the technology are increase rapidly, the currents of motor drives are controlled by vector controlled and the motor speed is estimated by the observer. The Observer is used to estimate the speed of motor drives usually in alfa-beta axis, and this situation will be more complicated if want to add a compensator, because the flux model, controller, and the decoupling equations are in dq-axis.

This paper proposed a new method to move the observer to dq-axis. The motor drives usually in three-phase system, and the mathematic models that’s developed is in two-phase because the calculation and analysis will be simpler [3]. The Clarke transformation is used for transformation from three-phase model into stationary two-phase model, or alfa-beta axis.

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & -1 \\
1 & -1 & -1 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_o
\end{bmatrix}
\]

(1.1)

Transformation from stationary two-phase \(\alpha-\beta\) axis into rotating two-phase model dq axis used Park transformation is shown in Eqs. (1.2)

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_e & \sin \theta_e \\
-\sin \theta_e & \cos \theta_e
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b
\end{bmatrix}
\]

(1.2)

The matrices transformation from three-phase model into rotating two-phase model is:

\[
\begin{bmatrix}
i_d \\
i_q \\
i_o
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_e & \cos(\theta_e - \frac{2\pi}{3}) & \cos(\theta_e + \frac{2\pi}{3}) \\
\frac{2}{3} & -\sin \theta_e & -\sin(\theta_e - \frac{2\pi}{3}) & -\sin(\theta_e + \frac{2\pi}{3}) \\
1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

(1.3)

II. METHODOLOGY

A. The Actual Models of Induction Motor

The general equations for induction motor drives are [2]:

\[
\ddot{\psi}_s = R_s \dot{\psi}_s + \frac{d}{dt}\dot{\psi}_s + j \omega_s \psi_s
\]

(2.1)
\[ \ddot{V}_r = R_r \dot{i}_r + \frac{d}{dt} \psi_r + j(\omega_c - \omega_r) \dot{\psi}_r \]  
(2.2)

\[ \dot{\psi}_r = L_r \dot{i}_r + L_m \dot{i}_r \]  
(2.3)

\[ \ddot{\psi}_r = L_r \dot{i}_r + L_m \dot{i}_r \]  
(2.4)

Equations (2.1) until (2.4) are used to get the mathematic models of induction motor drives in alfa-beta axis, so that \( \omega_c \) is zero. The rotor type is squirrel cage, then rotor voltage \( V_r \) is zero. The mathematic model of induction motor drives in alfa-beta axis are:

\[ \frac{d}{dt} i_{\alpha s} = \frac{1}{\sigma L_s} V_{\alpha s} + \left(- \frac{R_s}{\sigma L_s} - \frac{(1-\sigma)}{\sigma L_s} \right) i_{\alpha s} + \frac{L_m}{\sigma L_s L_r} \psi_{\alpha s} + \frac{L_m}{\sigma L_r} \psi_{\beta s} \]  
(2.5)

\[ \frac{d}{dt} i_{\beta s} = \frac{1}{\sigma L_s} V_{\beta s} + \frac{L_m}{\sigma L_s L_r} \psi_{\alpha s} - \frac{L_m}{\sigma L_r} \psi_{\beta s} \]  
(2.6)

\[ \frac{d}{dt} \psi_{\alpha s} = - \frac{R_s}{L_s} \psi_{\alpha s} + \frac{R_r}{L_r} L_m i_{\alpha s} - \omega_r \psi_{\beta s} \]  
(2.7)

\[ \frac{d}{dt} \psi_{\beta s} = - \frac{R_r}{L_r} \psi_{\beta s} + \frac{R_r}{L_r} L_m i_{\beta s} + \omega_r \psi_{\alpha s} \]  
(2.8)

with \( \sigma = \frac{L_r L_s - L_m^2}{L_r L_s} \) and \( \tau_r = \frac{L_r}{R_r} \)

The stator voltage equations:

\[ v_{\alpha s} = R_s i_{\alpha s} + L_s \sigma \frac{d}{dt} i_{\alpha s} - \omega_r L_s \sigma i_{\beta s} + L_s \left(1 - \sigma\right) \frac{d}{dt} i_{\gamma s} \]  
(2.9)

\[ v_{\beta s} = R_s i_{\beta s} + L_s \sigma \frac{d}{dt} i_{\beta s} + \omega_r L_s \sigma i_{\alpha s} + L_s \left(1 - \sigma\right) \omega_r i_{\gamma s} \]  
(2.10)

The PI controller is used to control the current vector, but this controller can only control a linear system, so equations (2.9) and (2.10) must be linearized first by the decoupling.

\[ v_{\alpha d} = u_{\alpha d} + v_{\alpha d} \]  
(2.11)

\[ v_{\alpha q} = u_{\alpha q} + v_{\alpha q} \]  
(2.12)

\[ u_{\alpha d} = R_s i_{\alpha d} + L_s \sigma \frac{d}{dt} i_{\alpha d} \]  
(2.13)

\[ u_{\alpha q} = R_s i_{\alpha q} + L_s \sigma \frac{d}{dt} i_{\alpha q} \]  
(2.14)

\[ v_{\gamma d} = -\omega_r L_s \sigma i_{\gamma q} + L_s \left(1 - \sigma\right) \frac{d}{dt} i_{\gamma q} \]  
(2.15)

\[ v_{\gamma q} = \omega_r L_s \sigma i_{\gamma d} + L_s \left(1 - \sigma\right) \omega_r i_{\gamma q} \]  
(2.16)

\( v_{\alpha d} \) and \( v_{\alpha q} \) are the coupling voltages, \( u_{\alpha d} \) and \( u_{\alpha q} \) are the stator voltages after decoupling process.

The flux model equations are:

\[ \frac{d}{dt} i_{\alpha r} = \frac{R_r}{L_r} \left(i_{\alpha d} - i_{\alpha r} \right) \]  
(2.17)

\[ \omega_c = p \omega_r + \frac{R_r}{L_r} i_{\gamma s} \]  
(2.18)

\[ \frac{d}{dt} \theta_v = \omega_c \]  
(2.19)

and the rotor speed equation is [8]:

\[ \frac{d}{dt} \omega_r = \frac{(T_v - T_r)}{J} \]  
(2.20)

The torque equation is:

\[ T_v = N \frac{L_m}{L_r} i_{\gamma s} \psi_{\alpha r} = N \frac{L_m^2}{L_r} i_{\gamma s} i_{\gamma r} = N(1 - \sigma)L_s i_{\gamma s} i_{\gamma r} \]  
(2.21)

B. PI Controller and PWM

The d-axis stator current, \( i_{\alpha d} \) fed to the system is constant at 2 ampere and the q-axis stator current, \( i_{\alpha q} \) has variation in the value, with maximum value is 3 ampere, minimum value -1 ampere and stable in 0.2 ampere. The PI controller is used in this simulation.

From the decoupling voltages equation and general equation of PI controller, we get the equation for the direct axis is:

\[ k_{\alpha d} = \frac{L_s \sigma}{T_d} \]  
(2.22)

and for the quadrature axis is:

\[ k_{\alpha q} = \frac{L_s \sigma}{T_d} \]  
(2.23)

The signal from PI controller will be given to Pulse Width Modulation (PWM) before fed the actual induction motor model. PWM is used to transform sinusoid wave into discrete pulse with width of the pulse different between one another depend on the sinusoid wave as an input.
C. Full Order Observer in dq-axis

The Observer is used to estimate the value of current, flux and speed of the motor. This observer is moved to dq-axis, so the \( \omega_e \) value is not same as zero anymore. The reference frame can be viewed in the Fig. 1 below :

Models of the system describe in state space equation are:

\[
\dot{x} = Ax + Bu \quad y = Cx \tag{2.24}
\]

Matrices A and C are observable, so from the output, the state of the system can be known \[4\]. The observer equations are :

\[
\hat{x} = A\hat{x} + Bu + G(y - \hat{y}) \quad \hat{y} = C\hat{x} \tag{2.25}
\]

The ‘^\hat{}’ sign indicates the estimated value. A is the motor models matrices in dq-axis that can be gotten from equations (2.1) – (2.4), because the motor model is in dq-axis, the \( \omega_e \) value is not zero. The motor models in state space is :

\[
\begin{bmatrix}
R + L \frac{1}{\tau_c} & \frac{L}{\tau_\psi} & \frac{L_\theta}{\tau_\psi} & 0 \\
\frac{L}{\tau_\psi} & \frac{1}{\tau} & 0 & 0 \\
\frac{L_\theta}{\tau_\psi} & 0 & \frac{1}{\tau} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} i_n \\
\frac{d}{dt} i_q \\
\frac{d}{dt} \psi_q \\
\frac{d}{dt} \psi_d
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
i_n \cr i_q \cr \psi_q \cr \psi_d
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} i_n \\
\frac{d}{dt} i_q \\
\frac{d}{dt} \psi_q \\
\frac{d}{dt} \psi_d
\end{bmatrix}
+ \begin{bmatrix}
R + L \frac{1}{\tau_c} & \frac{L}{\tau_\psi} & \frac{L_\theta}{\tau_\psi} & 0 \\
\frac{L}{\tau_\psi} & \frac{1}{\tau} & 0 & 0 \\
\frac{L_\theta}{\tau_\psi} & 0 & \frac{1}{\tau} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
in \\
 iq \\
\psi_d \\
\psi_q
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} i_n \\
\frac{d}{dt} i_q \\
\frac{d}{dt} \psi_q \\
\frac{d}{dt} \psi_d
\end{bmatrix}
\]

\[
\dot{x} = Ax + Bu + G(y - \hat{y}) \tag{2.26}
\]

To get the observer gain, the motor models in dq-axis with state space equation is used and shown in the following equations :

\[
\frac{d}{dt} \dot{x} = Ax + Bu + G(y - \hat{y}) \tag{2.28}
\]

With \( G = \begin{bmatrix} gL + gJ \cr gL + gJ \end{bmatrix} \), \( I = \begin{bmatrix} 1 & 0 \cr 0 & 1 \end{bmatrix} \) and \( J = \begin{bmatrix} 0 & -1 \cr 1 & 0 \end{bmatrix} \)

The eigen value from motor models is \( \mu \) and the eigen value from observer models is \( \lambda \) and take assume that observer model’s eigen value is \( k \) times higher than eigen value from motor models.

With identity equations, to get the observer gain with assumption that \( \hat{\omega}_e = \omega_e \):

\[
\begin{align*}
g_1 &= \frac{(k - 1)}{k} \left[ -\frac{R}{\sigma L} - \frac{R}{\sigma L} \right] \\
g_2 &= -\frac{(k - 1)}{k} (\hat{\omega}_e) \\
g_3 &= \frac{(k - 1)}{k(\hat{\omega}_e, \tau_e)} \left( \frac{R, R, \tau_e, L, \sigma L, \sigma \tau_e, \tau_e, \tau_e}{L_m} \right) \\
g_4 &= \frac{(k - 1)}{k(\hat{\omega}_e, \tau_e)} \left( \frac{(R, L, \tau_e, \tau_e, L, \sigma L, L, \tau_e, \tau_e, \tau_e}{L_m} \right)
\end{align*}
\]

The equations of induction motor models with full order observer are :

\[
\begin{align*}
\frac{d}{dt} \frac{1}{L_i} V_{ad} + \left( \frac{R}{L_i} \right) \frac{d}{dt} \frac{1}{L_i} V_{ad} + \frac{L}{L_i} \psi_a + g_1 (-q_i - \dot{q}_i) &= \frac{L_\theta}{L_i} \psi_q + \frac{L_\theta}{L_i} \psi_q + g_1 (-q_i - \dot{q}_i) \\
\frac{d}{dt} \frac{1}{L_i} V_{ad} + \left( \frac{R}{L_i} \right) \frac{d}{dt} \frac{1}{L_i} V_{ad} + \frac{L}{L_i} \psi_a + g_1 (-q_i - \dot{q}_i) &= \frac{L_\theta}{L_i} \psi_q + \frac{L_\theta}{L_i} \psi_q + g_1 (-q_i - \dot{q}_i)
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dt} \psi_{ad} &= -\frac{L_\theta}{L_i} \psi_{ad} + \frac{L_\theta}{L_i} \psi_{ad} + \left( \psi_a - \dot{\psi}_a \right) \psi_a + g_1 (i_n - \dot{i}_n) - g_1 (i_n - \dot{i}_n) \\
\frac{d}{dt} \psi_{ad} &= -\frac{L_\theta}{L_i} \psi_{ad} + \frac{L_\theta}{L_i} \psi_{ad} + \left( \psi_a - \dot{\psi}_a \right) \psi_a + g_1 (i_n - \dot{i}_n) + g_1 (i_n - \dot{i}_n)
\end{align*}
\]

To estimate the speed, the Lyapunov theory is used. The Lyapunov function candidate is described as \[8\] :

\[
\begin{align*}
\frac{d}{dt} \psi_{ad} &= -\frac{L_\theta}{L_i} \psi_{ad} + \frac{L_\theta}{L_i} \psi_{ad} + \left( \psi_a - \dot{\psi}_a \right) \psi_a + g_1 (i_n - \dot{i}_n) - g_1 (i_n - \dot{i}_n) \\
\frac{d}{dt} \psi_{ad} &= -\frac{L_\theta}{L_i} \psi_{ad} + \frac{L_\theta}{L_i} \psi_{ad} + \left( \psi_a - \dot{\psi}_a \right) \psi_a + g_1 (i_n - \dot{i}_n) + g_1 (i_n - \dot{i}_n)
\end{align*}
\]
\[ V = e^T e + \frac{(\dot{\omega}_r - \omega_r)^2}{\delta} \]  

(2.36)

According to the Lyapunov theory, the system will be stable if Lyapunov function candidate, \( V \), as Eqs. (2.36) [5], and the derivation of \( V \) is shown in Eq. (2.37) is also negative semidefinite to.

\[
\frac{d}{dt} V = e^T [(A - GC)^T + (A - GC)]e \\
\quad - 2 \frac{\Delta \omega}{c}(\dot{\psi}_{rq} e_{rq} - \dot{\psi}_{rd} e_{rd}) \\
\quad + 2 \frac{\Delta \omega_r}{\delta} \frac{d}{dt} \omega_r,
\]

(2.37)

The \( G \) matrix is negative semidefinite, then the \([(A - GC)^T + (A - GC)]\) matrix is also negative semidefinite. So the derivative of \( V \) will be negative semidefinite if the second and third row of the equation (2.46) are zero, the estimate speed is shown in Eq. (2.38).

\[
\dot{\omega}_r = K_p (\dot{\psi}_{rq} e_{rq} - \dot{\psi}_{rd} e_{rd}) + K_f \int (\dot{\psi}_{rq} e_{rq} - \dot{\psi}_{rd} e_{rd}) dt
\]

(2.38)

The proportional component \( P \) is added to reduce the steady state error.

III. RESULTS

A. System with full order observer :

The parameters’s motor that is used in this simulation as below [8]:

\( L_m = 0.2279 \) H, \( L_r = 0.2349 \) H, \( L_s = 0.2349 \) H; \( P = 750 \) watt \( R_s = 2.76 \) \( \Omega \), \( R_r = 2.90 \) \( \Omega \), pole = 4, Sampling time = \( 10^{-4} \) s

Block diagram of the system with full order observer is shown in Fig 2:

The simulation results for :

\( k_{observer} : 0.75, k_p : 8 \) and \( k_i : 650 \)

Fig 3. Stator current d-axis, \( i_{sd} \)

Fig 4. Stator current q-axis, \( i_{sq} \)

Fig 5. Rotor flux d-axis, \( \psi_{rd} \)

Fig 6. Rotor flux q-axis, \( \psi_{rq} \)

Fig 2. Block diagram with full order observer
The all graphics describe that between the actual and the estimated value are almost the same, although in some graphics some ripples are found. The difference between them is caused by the difference of the \( \theta_e \), because in dq-axis, \( \theta_e \) becomes a crucial factor.

In the beginning of the magnetizing current Fig. 8, an overshoot that caused by the characteristics of the motor can be seen, and \( \theta_e \) between the actual and the estimated is different. The different between \( \theta_e \) actual and estimated is found until 3 seconds, and after 3 seconds the overshoot is not exist anymore. The rotor speed error between the actual and the estimated is 0.2297% and the torque is 3.1488%.

**B. Compensator in Full Order Observer**

The compensator equations are:

\[
\frac{d}{dt}i_{sr}^{\text{comp}} = \frac{R}{L_r}(i_{sr} - i_{sr}^{\text{comp}} + k_{error}e_{srd}) \quad (3.1)
\]

\[
\omega_e^{\text{comp}} = \omega_e + \frac{R}{L_r} \left( i_{sq} - k_{error}e_{sq} \right) \quad (3.2)
\]

Block diagram of the systems is shown as below:

![Block diagram with compensator in full order observer](image)

The simulation results are shown in the following Figure:

- \( k_{observer} : 0.75, k_{error} : 0.7, k_p : 8 \) and \( k_i = 650 \),

![Rotor flux d-axis, \( i_{rd} \)](image)

System with compensator describes that there isn’t overshoot in magnetizing current anymore and the settling time of the system is to be shorter than before. The rotor speed error between the actual and the estimated is 0.1081 % and for the torque is 1.4308 %.

To analyze the error, the q-axis input current $i_{sq}^*$ changed into step, with maximum value is 0.8 ampere and the minimum value is zero and the d-axis input current, $i_{sd}^*$ is same as before. And the results are shown as bellow:

**CONCLUSIONS**

1. The difference induction motor model between actual and estimated is able to be used.
2. With observer in dq-axis, the dq current compensator can be used less complicated.
3. All the value of estimated parameters are affect by $\theta_e$, so that $\theta_e$ is an important factor in the whole system.
5. The speed estimation can follow speed actual begin 100 rpm until 1150 rpm, after that the speed has very small fluctuated , rotor speed error is 0.2297 %
6. The dq current compensator in full order observer gives a significant result , the rotor speed error between actual and estimated is 0.1081 %.
7. The torque error between actual and estimate is 1.4308 %.
8. If the input current $i_{sq}^*$ changed into step, the compensator error in full order observer becomes bigger.

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