Controller Design via Root Locus
The Industrial Operations Hierarchy

- Supervision
  - Management, Coordination, Optimization
    - Estimator management
    - Controller management
    - Fault detection
    - Fault diagnose
    - Fault isolation
    - Adjustment mechanism
    - Control algorithm: PA, MPC, deadbeat, MV, Fuzzy, NN, Ackermann

- Control
  - Process 1
  - Process 2
Analysis and Control Objectives

- Improving steady state error
- Improving transient response
- Stability
- Low cost
- Robustness
<table>
<thead>
<tr>
<th>Function</th>
<th>Compensator</th>
<th>Transfer function</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| Improve steady-state error| PI          | $\frac{Ks + z_c}{s}$ | 1. Increases system type.  
2. Error becomes zero.  
3. Zero at $-z_c$ is small and negative.  
4. Active circuits are required to implement. |
| Improve steady-state error| Lag         | $\frac{Ks + z_c}{s + p_c}$ | 1. Error is improved but not driven to zero.  
2. Pole at $-p_c$ is small and negative.  
3. Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$.  
4. Active circuits are not required to implement. |
| Improve transient response| PD          | $K(s + z_c)$ | 1. Zero at $-z_c$ is selected to put design point on root locus.  
2. Active circuits are required to implement.  
3. Can cause noise and saturation; implement with rate feedback or with a pole (lead). |
| Improve transient response| Lead        | $\frac{Ks + z_c}{s + p_c}$ | 1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus.  
2. Pole at $-p_c$ is more negative than zero at $-z_c$.  
3. Active circuits are not required to implement. |
<table>
<thead>
<tr>
<th>Function</th>
<th>Compensator</th>
<th>Transfer function</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improve steady-state error and transient response</td>
<td>$K\frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{s}$</td>
<td>1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error.</td>
<td>2. Lead zero at $-z_{\text{lead}}$ improves transient response.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, the origin.</td>
<td>4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Active circuits required to implement.</td>
<td>6. Can cause noise and saturation; implement with rate feedback or with an additional pole.</td>
</tr>
<tr>
<td>Peg-lead</td>
<td>$K\frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{(s + p_{\text{lag}})(s + p_{\text{lead}})}$</td>
<td>1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error.</td>
<td>2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Lag pole at $-p_{\text{lag}}$ is small and negative.</td>
<td>4. Lag zero at $-z_{\text{lag}}$ is close to, and to the left of, lag pole at $-p_{\text{lag}}$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus.</td>
<td>6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7. Active circuits are not required to implement.</td>
<td></td>
</tr>
</tbody>
</table>
The Relay Experiment

Autotuning PID Controller
Self-Tuning Control System Scheme
(2-DOF Controller)

Performance requirements

Configuration requirements

Controller synthesis

Controller parameters; H, F, G

Recursive estimator

Estimated system parameters; A, B

Pole assignment

Deadbeat

Adjustment mechanism

PRBS Generator

Pressure process rig

2-DOF controller

Output signal y(t)

Control signal u(t)

Reference signal w(t)

RLS

RIV
Figure 9.1

a. Sample root locus, showing possible design point via gain adjustment (A) and desired design point that cannot be met via simple gain adjustment (B);
b. responses from poles at A and B
Figure 9.2
Compensation techniques:

a. cascade;
b. feedback
Figure 9.3
Pole at A is:
a. on the root locus without compensator;
b. not on the root locus with compensator pole added;
(figure continues)

\[-\theta_1 - \theta_2 - \theta_3 = (2k + 1)180^\circ\]  \hspace{1cm} \text{(a)}

\[-\theta_1 - \theta_2 - \theta_3 - \theta_c \neq (2k + 1)180^\circ\]  \hspace{1cm} \text{(b)}
Figure 9.3  
(continued)  
c. approximately on the root locus with compensator pole and zero added

\[-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k + 1)180^\circ\]
Figure 9.4
Closed-loop system for Example 9.1:
(a) before compensation;
(b) after ideal integral compensation
Figure 9.5
Root locus for uncompensated system of Figure 9.4(a)

\[ K = 164.6 \]

\[ \zeta = 0.174 \]

\[ -0.694 + j3.926 \]

\[ j2 \]

\[ 100.02^\circ \]

\[ X = \text{Closed-loop pole} \]

\[ X = \text{Open-loop pole} \]
Figure 9.6
Root locus for compensated system of Figure 9.4(b)

$\zeta = 0.174$

$-0.678 + j3.837$

$K = 158.2$

$100.02^\circ$

$\ast = $ Closed-loop pole

$\ast = $ Open-loop pole

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Figure 9.7
Ideal integral compensated system response and the uncompensated system response of Example 9.1
Figure 9.8
PI controller

\[ \frac{K_2}{s} \]

\[ K_1 \]

Plant

\[ G(s) \]

\[ R(s) \]

\[ C(s) \]
Lag Compensation

**Figure 9.9**

a. Type 1 uncompensated system;
b. Type 1 compensated system;
c. compensator pole-zero plot

(a) $G(s) = \frac{K(s + z_c)}{s + p_c}$

(b) $G_c(s) = \frac{(s + z_c)}{(s + p_c)}$

(c) $s$-plane plot with compensator zeros and poles.
Figure 9.10
Root locus:

a. before lag compensation;
b. after lag compensation
Figure 9.11
Compensated system
for Example 9.2
Figure 9.12
Root locus for compensated system of Figure 9.11

- X = Closed-loop pole
- = Open-loop pole

\( \zeta = 0.174 \)

\(-0.678 + j3.836 \)

\( K = 158.1 \)

Compensator pole at \(-0.01\)

Fourth closed-loop pole at \(-0.101\)
### Table 9.1
Predicted characteristics of uncompensated and lag-compensated systems for Example 9.2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncompensated</th>
<th>Lag-compensated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant and compensator</td>
<td>$\frac{K}{(s + 1)(s + 2)(s + 10)}$</td>
<td>$\frac{K(s + 0.111)}{(s + 1)(s + 2)(s + 10)(s + 0.01)}$</td>
</tr>
<tr>
<td>$K$</td>
<td>164.6</td>
<td>158.1</td>
</tr>
<tr>
<td>$K_p$</td>
<td>8.23</td>
<td>87.75</td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.108</td>
<td>0.011</td>
</tr>
<tr>
<td>Dominant second-order poles</td>
<td>$-0.694 \pm j3.926$</td>
<td>$-0.678 \pm j3.836$</td>
</tr>
<tr>
<td>Third pole</td>
<td>$-11.61$</td>
<td>$-11.55$</td>
</tr>
<tr>
<td>Fourth pole</td>
<td>None</td>
<td>$-0.101$</td>
</tr>
<tr>
<td>Zero</td>
<td>None</td>
<td>$-0.111$</td>
</tr>
</tbody>
</table>
Figure 9.13
Step responses of uncompensated and lag-compensated systems for Example 9.2
Figure 9.14
Step responses of the system for Example 9.2 using different lag compensators

\[ G_c(s) = \frac{s + 0.111}{s + 0.01} \]

\[ G_c(s) = \frac{s + 0.0111}{s + 0.001} \]
Improving Transient Response

Figure 9.15
Using ideal derivative compensation:
a. uncompensated; b. compensator zero at –2;
Figure 9.15 (continued)
c. compensator zero at $-3$; d. compensator zero at $-4$
Figure 9.16
Uncompensated system and ideal derivative compensation solutions from Table 9.2
### Table 9.2
Predicted characteristics for the systems of Figure 9.15

<table>
<thead>
<tr>
<th>Plant and compensator</th>
<th>Uncompensated</th>
<th>Compensation b</th>
<th>Compensation c</th>
<th>Compensation d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dom. poles</td>
<td>$-0.939 \pm j2.151$</td>
<td>$-3 \pm j6.874$</td>
<td>$-2.437 \pm j5.583$</td>
<td>$-1.869 \pm j4.282$</td>
</tr>
<tr>
<td>$K$</td>
<td>23.72</td>
<td>51.25</td>
<td>35.34</td>
<td>20.76</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>2.347</td>
<td>7.5</td>
<td>6.091</td>
<td>4.673</td>
</tr>
<tr>
<td>%$OS$</td>
<td>25.38</td>
<td>25.38</td>
<td>25.38</td>
<td>25.38</td>
</tr>
<tr>
<td>$T_s$</td>
<td>4.26</td>
<td>1.33</td>
<td>1.64</td>
<td>2.14</td>
</tr>
<tr>
<td>$T_p$</td>
<td>1.46</td>
<td>0.46</td>
<td>0.56</td>
<td>0.733</td>
</tr>
<tr>
<td>$K_p$</td>
<td>2.372</td>
<td>10.25</td>
<td>10.6</td>
<td>8.304</td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.297</td>
<td>0.089</td>
<td>0.086</td>
<td>0.107</td>
</tr>
<tr>
<td>Third pole</td>
<td>$-6.123$</td>
<td>None</td>
<td>$-3.127$</td>
<td>$-4.262$</td>
</tr>
<tr>
<td>Zero</td>
<td>None</td>
<td>None</td>
<td>$-3$</td>
<td>$-4$</td>
</tr>
<tr>
<td>Comments</td>
<td>Second-order approx. OK</td>
<td>Pure second-order</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
</tr>
</tbody>
</table>
Figure 9.17
Feedback control system for Example 9.3
Figure 9.18
Root locus for uncompensated system shown in Figure 9.17

$\zeta = 0.504$

$-1.205 + j2.064$

$K = 43.35$

$120.26^\circ$

$X = \text{Closed-loop pole}$

$X = \text{Open-loop pole}$
Table 9.3
Uncompensated and compensated system characteristics for Example 9.3

<table>
<thead>
<tr>
<th>Plant and compensator</th>
<th>Uncompensated</th>
<th>Simulation</th>
<th>Compensated</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{K}{s(s + 4)(s + 6)}$</td>
<td>$\frac{K(s + 3.006)}{s(s + 4)(s + 6)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dominant poles</td>
<td>$-1.205 \pm j2.064$</td>
<td></td>
<td>$-3.613 \pm j6.193$</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>43.35</td>
<td></td>
<td>47.45</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.504</td>
<td></td>
<td>0.504</td>
<td></td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>2.39</td>
<td></td>
<td>7.17</td>
<td></td>
</tr>
<tr>
<td>%OS</td>
<td>16</td>
<td>14.8</td>
<td>16</td>
<td>11.8</td>
</tr>
<tr>
<td>$T_s$</td>
<td>3.320</td>
<td>3.6</td>
<td>1.107</td>
<td>1.2</td>
</tr>
<tr>
<td>$T_p$</td>
<td>1.522</td>
<td>1.7</td>
<td>0.507</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_v$</td>
<td>1.806</td>
<td></td>
<td>5.94</td>
<td></td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.554</td>
<td></td>
<td>0.168</td>
<td></td>
</tr>
<tr>
<td>Third pole</td>
<td>$-7.591$</td>
<td></td>
<td>$-2.775$</td>
<td></td>
</tr>
<tr>
<td>Zero</td>
<td>None</td>
<td></td>
<td>$-3.006$</td>
<td></td>
</tr>
<tr>
<td>Comments</td>
<td>Second-order</td>
<td></td>
<td>Pole-zero</td>
<td>not canceling</td>
</tr>
<tr>
<td></td>
<td>approx. OK</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 9.19
Compensated dominant pole superimposed over the uncompensated root locus for Example 9.3

$\zeta = 0.504$

$-3.613 + j6.193$

Desired compensated dominant pole

Uncompensated dominant pole $-1.205 + j2.064$

$\sigma$ = Closed-loop pole

$\mathbf{X}$ = Open-loop pole

$s$-plane

$120.26^\circ$
Figure 9.20
Evaluating the location of the compensating zero for Example 9.3

$\zeta = 0.504$

$-3.613 + j6.193$

Desired compensated dominant pole

Uncompensated dominant pole

$-1.205 + j2.064$

$95.6^\circ$

$120.26^\circ$

$\times = $ Closed-loop pole

$X = $ Open-loop pole
Figure 9.21
Root locus for the compensated system of Example 9.3

\[ \zeta = 0.504 \]
\[ -3.613 + j6.193 \]
\[ K = 47.45 \]

Compensated dominant pole

\[ -2.775 \]

\( X = \text{Closed-loop pole} \)
\( X = \text{Open-loop pole} \)

\( 120.26^\circ \)
Figure 9.22
Uncompensated and compensated system step responses of Example 9.3
Figure 9.23
PD controller

\[ R(s) \rightarrow K_1 \rightarrow G(s) \rightarrow C(s) \]

\[ K_2s \]

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Figure 9.24
Geometry of lead compensation
Figure 9.25
Three of the infinite possible lead compensator solutions
Figure 9.26
Lead compensator design, showing evaluation of uncompensated and compensated dominant poles for Example 9.4

\[ \zeta = 0.358 \]

Desired compensated dominant pole

\[-2.014 + j5.252 \]

Uncompensated dominant pole

\[-1.007 + j2.627 \]

\[ K = 63.21 \]

\[ 110.98^\circ \]

\[ \sigma \]

\[ j \omega \]

\[ j6 \]

\[ j5 \]

\[ j4 \]

\[ j3 \]

\[ s-plane \]

\[ j1 \]

\[ 0 \]

\[ \times \] = Closed-loop pole

\[ \times \] = Open-loop pole
### Table 9.4
Comparison of lead compensation designs for Example 9.4

<table>
<thead>
<tr>
<th></th>
<th>Uncompensated</th>
<th>Compensation a</th>
<th>Compensation b</th>
<th>Compensation c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant and compensator</td>
<td>$\frac{K}{s(s + 4)(s + 6)}$</td>
<td>$\frac{K(s + 5)}{(s(s + 4)(s + 6)(s + 42.96)}$</td>
<td>$\frac{K(s + 4)}{(s(s + 4)(s + 6)(s + 20.09)}$</td>
<td>$\frac{K(s + 2)}{(s(s + 4)(s + 6)(s + 8.971)}$</td>
</tr>
<tr>
<td>Dominant poles</td>
<td>$-1.007 \pm j2.627$</td>
<td>$-2.014 \pm j5.252$</td>
<td>$-2.014 \pm j5.252$</td>
<td>$-2.014 \pm j5.252$</td>
</tr>
<tr>
<td>$K$</td>
<td>63.21</td>
<td>1423</td>
<td>698.1</td>
<td>345.6</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.358</td>
<td>0.358</td>
<td>0.358</td>
<td>0.358</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>2.813</td>
<td>5.625</td>
<td>5.625</td>
<td>5.625</td>
</tr>
<tr>
<td>$% OS^*$</td>
<td>30 (28)</td>
<td>30 (30.7)</td>
<td>30 (28.2)</td>
<td>30 (14.5)</td>
</tr>
<tr>
<td>$T_s^*$</td>
<td>3.972 (4)</td>
<td>1.986 (2)</td>
<td>1.986 (2)</td>
<td>1.986 (1.7)</td>
</tr>
<tr>
<td>$T_p^*$</td>
<td>1.196 (1.3)</td>
<td>0.598 (0.6)</td>
<td>0.598 (0.6)</td>
<td>0.598 (0.7)</td>
</tr>
<tr>
<td>$K_v$</td>
<td>2.634</td>
<td>6.9</td>
<td>5.791</td>
<td>3.21</td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.380</td>
<td>0.145</td>
<td>0.173</td>
<td>0.312</td>
</tr>
<tr>
<td>Other poles</td>
<td>$-7.986$</td>
<td>$-43.8, -5.134$</td>
<td>$-22.06$</td>
<td>$-13.3, -1.642$</td>
</tr>
<tr>
<td>Zero</td>
<td>None</td>
<td>$-5$</td>
<td>None</td>
<td>$-2$</td>
</tr>
<tr>
<td>Comments</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
<td>No pole-zero cancellation</td>
</tr>
</tbody>
</table>

* Simulation results are shown in parentheses.
Figure 9.27
s-plane picture used to calculate the location of the compensator pole for Example 9.4

Note: This figure is not drawn to scale.
Figure 9.28
Compensated system root locus

Note: This figure is not drawn to scale.
Figure 9.29
Uncompensated system and lead compensation responses for Example 9.4
PID Controller Design

Figure 9.30

PID controller
Figure 9.31
Uncompensated feedback control system for Example 9.5
Figure 9.32
Root locus for the uncompensated system of Example 9.5

\[ \zeta = 0.456 \]

\[ -5.415 + j10.57 \]

\[ K = 121.5 \]

Uncompensated dominant pole

\[ \text{X} = \text{Closed-loop pole} \]

\[ \text{X} = \text{Open-loop pole} \]
### Table 9.5
Predicted characteristics of uncompensated, PD-, and PID-compensated systems of Example 9.5

<table>
<thead>
<tr>
<th>Plant and compensator</th>
<th>Uncompensated</th>
<th>PD-compensated</th>
<th>PID-compensated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(s + 8)$</td>
<td>$\frac{K(s + 8)}{(s + 3)(s + 6)(s + 10)}$</td>
<td>$\frac{K(s + 8)(s + 55.92)}{(s + 3)(s + 6)(s + 10)}$</td>
<td>$\frac{K(s + 8)(s + 55.92)(s + 0.5)}{(s + 3)(s + 6)(s + 10)s}$</td>
</tr>
<tr>
<td>Dominant poles</td>
<td>$-5.415 \pm j10.57$</td>
<td>$-8.13 \pm j15.87$</td>
<td>$-7.516 \pm j14.67$</td>
</tr>
<tr>
<td>$K$</td>
<td>121.5</td>
<td>5.34</td>
<td>4.6</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.456</td>
<td>0.456</td>
<td>0.456</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>11.88</td>
<td>17.83</td>
<td>16.49</td>
</tr>
<tr>
<td>%OS</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_s$</td>
<td>0.739</td>
<td>0.492</td>
<td>0.532</td>
</tr>
<tr>
<td>$T_p$</td>
<td>0.297</td>
<td>0.198</td>
<td>0.214</td>
</tr>
<tr>
<td>$K_p$</td>
<td>5.4</td>
<td>13.27</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.156</td>
<td>0.070</td>
<td>0</td>
</tr>
<tr>
<td>Other poles</td>
<td>$-8.169$</td>
<td>$-8.079$</td>
<td>$-8.099, -0.468$</td>
</tr>
<tr>
<td>Zeros</td>
<td>$-8$</td>
<td>$-8, -55.92$</td>
<td>$-8, -55.92, -0.5$</td>
</tr>
<tr>
<td>Comments</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
<td>Zero at $-55.92$ and $-0.5$ not canceled</td>
</tr>
</tbody>
</table>
Figure 9.33
Calculating the PD compensator zero for Example 9.5

Note: This figure is not drawn to scale.
Figure 9.34
Root locus for PD-compensated system of Example 9.5

$\zeta = 0.456$

$-8.13 + j15.87$
$K = 5.34$

$117.13^\circ$

$X =$ Closed-loop pole
$\times =$ Open-loop pole

Note: This figure is not drawn to scale.
Figure 9.35
Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5
Figure 9.36
Root locus for PID-compensated system of Example 9.5

\[ \zeta = 0.456 \]
\[ -7.516 + j14.67 \quad K = 4.6 \]

\( s \)-plane

\( j\omega \)

\[ \text{PID-compensated dominant pole} \]

\( 117.13^\circ \)

\( \times = \text{Closed-loop pole} \)
\( \bigcirc = \text{Open-loop pole} \)

Note: This figure is not drawn to scale.
Lag-lead Compensator Design

Figure 9.37
Uncompensated system for Example 9.6

\[ R(s) + \frac{K}{s(s + 6)(s + 10)} - E(s) \]

\[ C(s) \]
Figure 9.38
Root locus for uncompensated system of Example 9.6
## Table 9.6
Predicted characteristics of uncompensated, lead-compensated, and lag-lead-compensated systems of Example 9.6

<table>
<thead>
<tr>
<th>Plant and compensator</th>
<th>Uncompensated</th>
<th>Lead-compensated</th>
<th>Lag-lead-compensated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(s + 6)(s + 10)$</td>
<td>$K$</td>
<td>$s(s + 10)(s + 29.1)$</td>
<td>$K(s + 0.04713)$</td>
</tr>
<tr>
<td>Dominant poles</td>
<td>$-1.794 \pm j3.501$</td>
<td>$-3.588 \pm j7.003$</td>
<td>$-3.574 \pm j6.976$</td>
</tr>
<tr>
<td>$K$</td>
<td>192.1</td>
<td>1977</td>
<td>1971</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.456</td>
<td>0.456</td>
<td>0.456</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>3.934</td>
<td>7.869</td>
<td>7.838</td>
</tr>
<tr>
<td>%OS</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_s$</td>
<td>2.230</td>
<td>1.115</td>
<td>1.119</td>
</tr>
<tr>
<td>$T_p$</td>
<td>0.897</td>
<td>0.449</td>
<td>0.450</td>
</tr>
<tr>
<td>$K_v$</td>
<td>3.202</td>
<td>6.794</td>
<td>31.92</td>
</tr>
<tr>
<td>$e(\infty)$</td>
<td>0.312</td>
<td>0.147</td>
<td>0.0313</td>
</tr>
<tr>
<td>Third pole</td>
<td>$-12.41$</td>
<td>$-31.92$</td>
<td>$-31.91, -0.0474$</td>
</tr>
<tr>
<td>Zero</td>
<td>None</td>
<td>None</td>
<td>$-0.04713$</td>
</tr>
<tr>
<td>Comments</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
<td>Second-order approx. OK</td>
</tr>
</tbody>
</table>
Figure 9.39
Evaluating the compensator pole for Example 9.6

\[ -p_c \]

\[ j \omega \]

\[ j7.003 \]

\[ -3.588 \]

\[ \sigma \]

\[ 15.35^\circ \]

\[ \times = \text{Closed-loop pole} \]

\[ \times = \text{Open-loop pole} \]
Figure 9.40
Root locus for lead-compensated system of Example 9.6

\[ \zeta = 0.456 \]

\[ -3.588 + j7.003 \]

\[ K = 1977 \]

Compensated dominant pole

117.13°

\( \sigma \)

\( j\omega \)

\( s\)-plane

\( -j9 \)

\( -j6 \)

\( -j3 \)

\( 0 \)

\( -33 \)

\( -30 \)

\( -27 \)

\( -24 \)

\( -21 \)

\( -18 \)

\( -15 \)

\( -12 \)

\( -9 \)

\( -6 \)

\( -3 \)

\( -31.91 \)

\( X = \text{Closed-loop pole} \)

\( \times = \text{Open-loop pole} \)
Figure 9.41
Root locus for lag-lead-compensated system of Example 9.6

X = Closed-loop pole
X = Open-loop pole

Note: This figure is not drawn to scale.
Figure 9.42
Improvement in step response for lag-lead-compensated system of Example 9.6
Figure 9.43
Improvement in ramp response error for the system of Example 9.6:

a. lead-compensated;

b. lag-lead-compensated