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## Multivariable Control of A 4x4 Process in Distillation Column

A. Wahid<sup>1</sup> and A. Ahmad<sup>2</sup>

1. Department of Chemical Engineering, University of Indonesia, Depok.  
2. Department of Chemical Engineering, Universiti Teknologi Malaysia, Johor  
Corresponding authors: [wahid@che.ui.edu](mailto:wahid@che.ui.edu) and [arshad@fkkksa.utm.my](mailto:arshad@fkkksa.utm.my)

### Abstract

*A model predictive control strategy is proposed for multivariable nonlinear control problem in a 4x4 process in distillation column. The aim is to provide a solution to nonlinear control problem that is favorable in terms of industrial implementation. The scheme utilizes multiple linear models to cover wider range of operating conditions. Depending on the operating conditions, suitable model is used in control computations. Servo and regulatory controls of the system are examined on constrained and unconstrained conditions. Comparisons are made to conventional controllers. The results confirmed the potentials of the proposed strategy.*

*Keywords – Tuning, model predictive control, multivariable, distillation column*

### I. Introduction

Predictive control is now one of the most widely used advanced control methods in industry, especially in the control of processes that are constrained, multivariable and uncertain. A large number of implementation algorithms, included industrial predictive control applications (Qin, 2003) have an appeared in the literature.

The cornerstone of MPC is the model (Clarke, 1996). It cause MPC is called MBPC (model-based predictive control). MPC uses models in 2 ways: using a reliable model to predict effect of past control moves on P (prediction horizon) future outputs, assuming no future moves, and using the same model to compute the optimal M (control horizon) controller moves. Implement first move and repeat procedure.

Dynamic matrix control (DMC) (Culter, 1980) is the most popular MPC algorithm used in the chemical process industry today. Over the past decade, DMC has been implemented on a wide range of process applications. A major part of DMC's appeal in industry stems from the use of a linear finite step response model of the process and a simple quadratic performance objective function. The objective function is minimized over a prediction horizon to compute the optimal controller output moves as a least-squares problem.

Tuning a controller is a direct way to reach its optimum performance. Tuning conventional controllers (P, PI, and PID) is related to obtain an optimum setting of controller parameters (controller gain  $K_c$ , integral time  $T_i$ , and derivative time  $T_d$ ). Ziegler-Nichols, Lopez, Ciancone, etc. (Marlin, 2000) are some examples of single-loop tuning in P, PI, and PID controllers. Huang, et al. (2003) has proposed a direct method for multi-loop (multivariable) PI/PID controller design based on FOPDT/SOPDT model of each loop.

Table 1 Non-adaptive DMC tuning strategy (Dougherty, 2003)

Approximate the process dynamics of all controller output to measured process variable pairs with FOPDT models:	
$\frac{y_r(s)}{u_r(s)} = \frac{K_{rs}e^{-\theta_{rs}s}}{\tau_{rs}s+1} \quad (r = 1, 2, \dots, R; s = 1, 2, \dots, S)$	
<ol style="list-style-type: none"> <li>1. Select the sample time as close as possible to:  <math display="block">T_{rs} = \text{Max}(0.1\tau_{rs}, 0.5\theta_{rs}) \quad (r = 1, 2, \dots, R; s = 1, 2, \dots, S)</math> <math display="block">T = \text{Min}(T_{rs})</math> </li> <li>2. Compute the prediction horizon, P, and the model horizon, N:  <math display="block">P = N = \text{Max}\left(\frac{3\tau_{rs}}{T} + k_{rs}\right) \text{ where } k_{rs} = \left(\frac{\theta_{rs}}{T} + 1\right) \quad (r = 1, 2, \dots, R; s = 1, 2, \dots, S)</math> </li> <li>3. Compute a control horizon, M:  <math display="block">M = \text{Max}\left(\frac{\tau_{rs}}{T} + k_{rs}\right) \quad (r = 1, 2, \dots, R; s = 1, 2, \dots, S)</math> </li> <li>4. Select the controlled variable weights, <math>y_r^2</math>, to scale process variable units to be the same.</li> <li>5. Compute the move suppression coefficients, <math>\lambda_r^2</math>:  <math display="block">\lambda_r^2 = \frac{M}{10} \sum_{s=1}^S \left[ y_r^2 K_{rs}^2 \left\{ P - k_{rs} - \frac{3\tau_{rs}}{2T} + 2 - \frac{(M-1)}{2} \right\} \right] \quad (s = 1, 2, \dots, S)</math> </li> <li>6. Implement DMC using the traditional step response matrix of the actual process and the initial values of the parameters computed in steps 1-6.</li> </ol>	



An MPC controller has certain parameters setting to achieve its optimum performance. Those parameters are sampling time (T), prediction horizon (P), model horizon (N), control horizon (M), controlled variable weights ( $\gamma_s^2$ ), and move suppression coefficients ( $\lambda_s^2$ ). During the time, trial-and-error efforts have been done to find out this goal until Shridhar & Cooper (Dougherty, 2003) proposed a tuning strategy for unconstraint SISO and multivariable MPC. Dougherty and Cooper (2003) proposed a non-adaptive DMC tuning strategy (see Table 1) based on all of FOPDT models in systems.

## II. A 4x4 Process of Distillation Column

### II.1 Alatiqi and Luyben 4x4 Process

The first 4x4 process is presented by Alatiqi and Luyben (1986). The system is the results of a quantitative study of the dynamics of two alternative distillation systems for separating ternary mixtures that contain small amounts (less than 20%) of the intermediate component in the feed. The process transfer function matrix of the complex sidestream column/stripper distillation process is given by

$$G(s) = \begin{bmatrix} \frac{4.09e^{-1.3s}}{(33s+1)(8.3s+1)} & \frac{-6.36e^{-0.2s}}{(31.6s+1)(20s+1)} & \frac{-0.25e^{-0.4s}}{21s+1} & \frac{-0.49e^{-5s}}{22s+1} \\ \frac{-4.17e^{-4s}}{45s+1} & \frac{6.93e^{-1.01s}}{44.6s+1} & \frac{-0.05e^{-5s}}{34.5s+1} & \frac{1.53e^{-2.8s}}{48s+1} \\ \frac{-1.73e^{-17s}}{13s+1} & \frac{5.11e^{-11s}}{13.3s+1} & \frac{4.61e^{-1.02s}}{18.5s+1} & \frac{-5.48e^{-0.5s}}{15s+1} \\ \frac{-11.18e^{-2.6s}}{(43s+1)(6.5s+1)} & \frac{14.04e^{-0.02s}}{(45s+1)(10s+1)} & \frac{-0.1e^{-0.05s}}{(31.6s+1)(5s+1)} & \frac{4.49e^{-0.6s}}{(48s+1)(6.3s+1)} \end{bmatrix} \quad (1)$$

The system contains four controlled variables and four manipulated variables. The four controlled variables are  $y_1$  (mole fraction of the first component in the distillate),  $y_2$  (mole fraction of the third component in the bottom),  $y_3$  (mole fraction of the second component in the sidestream), and  $y_4$  (temperature difference). Meanwhile, the four manipulated variables are  $u_1$  (reflux flowrate),  $u_2$  (main column reboiler heat-transfer rate),  $u_3$  (stripper reboiler heat-transfer rate), and  $u_4$  (liquid draw rate from main column to stripper). This system presents a challenging 4x4 multivariable control problem.

### II.2 Doukas and Luyben 4x4 Process

The second 4x4 process is presented by Doukas and Luyben (1978). They studied the dynamic of a distillation column producing a liquid sidestream product. The objective is to maintain four composition specifications on the product stream. The transfer function matrix for the 4x4 model is

$$G(s) = \begin{bmatrix} \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \frac{0.374e^{-7.75s}}{22.2s+1} & \frac{-9.811e^{-1.59s}}{11.36s+1} & \frac{-2.37e^{-27.33s}}{33.3s+1} \\ \frac{5.24e^{-60s}}{400s+1} & \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{5.984e^{-2.24s}}{14.29s+1} & \frac{0.422e^{-8.72s}}{(250s+1)^2} \\ \frac{-0.33e^{-0.68s}}{(2.38s+1)^2} & \frac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{2.38e^{-0.42s}}{(1.43s+1)^2} & 0.513e^{-s} \\ \frac{4.48e^{-0.52s}}{11.11s+1} & \frac{-0.176e^{-0.48s}}{(6.90s+1)^2} & \frac{-11.67e^{-1.91s}}{12.19s+1} & 15.54e^{-s} \end{bmatrix} \quad (2)$$

The controlled and manipulated variables are  $y_1$  (toluene impurity in the bottom),  $y_2$  (toluene impurity in the distillate),  $y_3$  (benzene impurity in the sidestream), and  $y_4$  (xyene impurity in the sidestream);  $u_1$  (sidestream flow rate),  $u_2$  (reflux ration),  $u_3$  (reboil duty), and  $u_4$  (side draw location).

Some methods to get the optimum performance of the plant have implemented. Luyben [8] have developed the direct method of PID controller was called BLT-1 based on Ziegler-Nichols method. Based on Luyben's BLT-1 controller, the new methods were called BLT-2, BLT-3, and BLT-4 [8] have proved their ability to reach a better performance than the previous method. Even, BLT-4 was better than DMC controller. Based on an effective open-loop (EOP) model, Huang et al. (2003) have improved BLTs methods especially in 2x2 process. But, in the 3x3 process the optimum performance is 56% for Huang et al. method and 44% for BLT-4. In the 4x4 process, Huang et al. is better than BLT, but not if compared by Lee et al. method. Huang et al. has 69% of the optimum results while Lee et al. has 31%.



Although, BLT was better than DMC (Monica, 1988), Dougherty and Cooper (2003) have proved that the applications of the non-adaptive DMC tuning strategy in the three 2x2 processes (general transfer function, multi-tank, and distillation column) have a satisfactory performance. So, we will implement this strategy to improve the control performance of two distillation columns that have 4x4 matrixes of input and output variables.

### III. Results and Discussions

#### III.1 Alatiqi and Luyben 4x4 Process

Alatiqi and Luyben process has 16 empirical models that consist of three FOPDT models, two models having zeros, and eleven SOPDT models. All of empirical models have to be changed in FOPDT model to calculate MPC controller tuning. We use some PRCs (process reaction curves) from step change testing of the SOPDTs and the models having zeros, and apply a method that is developed by Smith (Marlin, 2000) to obtain the FOPDT.

PI controller shows the poor performance (see Figure 1). Also, the controller performance as shown by Figure 2 is extremely poor, because the responses are unstable. So, in this case, the default setting can not be used in MPC controller. Inevitable, we have to use the tuning setting in this case.

Table 2 PID and DMC parameters setting in Alatiqi case 1 model (A1 4x4 process)

PI controller tuning (Lee et al. method)		DMC controller tuning
	$g_c(s) = k'_c \left( 1 + \frac{1}{\tau_R s} \right)$	Ts = 1.62 P = 191 M = 52
Loop 1:	,	
Loop 2:	,	
Loop 3:	,	
Loop 4:	,	

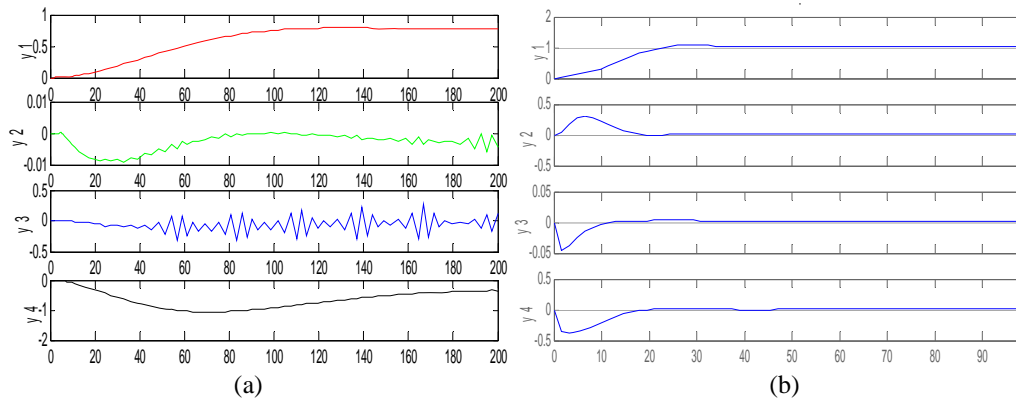


Figure 1 Responses of multi-loop control for alatiqi case 1 (A1 4x4) process: (a) PI controller performance using Lee et. al. tuning (unit step change in y1); (b) MPC controller performance using tuning setting Ts=1.62, P=191, M=52 (unit step change in y1)



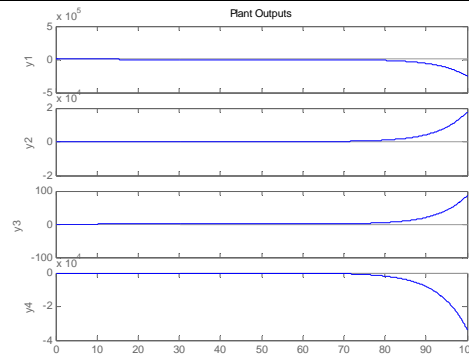


Figure 2 Alatiqi's Responses of MPC controller using default

### III.2 Doukas and Luyben 4x4 Process

Doukas and Luyben process has eight FOPDTs, six SOPDTs, and two dead times. The six SOPDT models are changed to FOPDT models in order to apply the DMC tuning. Because of the two dead times in the system (g34 and g44), two alternative parameters of the DMC tuning will be studied (Table 3). If the two dead times are involved in calculating the DMC tuning, it is an alternative 1. In the contrary, they are neglected, it is an alternative 2. However, the results presented too large parameters of the prediction (P) and control (M) horizon. They may appear some difficulties in calculation of MPC controller.

The huge DMC parameters are caused by the g24 transfer function. The approximated model of g24 has time constant of 410 time unit and dead time of 132 time unit. In additional, the sampling time (T) is enough little, while it is the de-numerator position as shown by Table 1. The alternative 2 present less than the alternative 1 is also caused by T.

Table 3 DMC parameter of Doukas-Luyben 4x4 process

DMC Parameter	Alternative 1	Alternative 2
Sampling Time (T)	0.5	0.6
Prediction Horizon (P)	4370	3673
Control Horizon (M)	1087	913

Before simulating the system, the two dead times excluded from the plant. Dead time only in the transfer function means that the input direct to output. This case must be avoided. Therefore, the plant of the 4x4 process is no g34 and g44 transfer functions.

Figure 3 show the performance of MPC controller using default parameter of  $T = 0.1$ ,  $P = 10$ , and  $M = 2$ . Meanwhile, MPC controller gives an error statement using DMC tuning parameter as shown by Table 3. Duogherty-Cooper's tuning strategy of DMC parameters present a poor performance when the constant time of FOPDT is too large. Other researchers should take this opportunity to follow up the study of DMC tuning strategy.



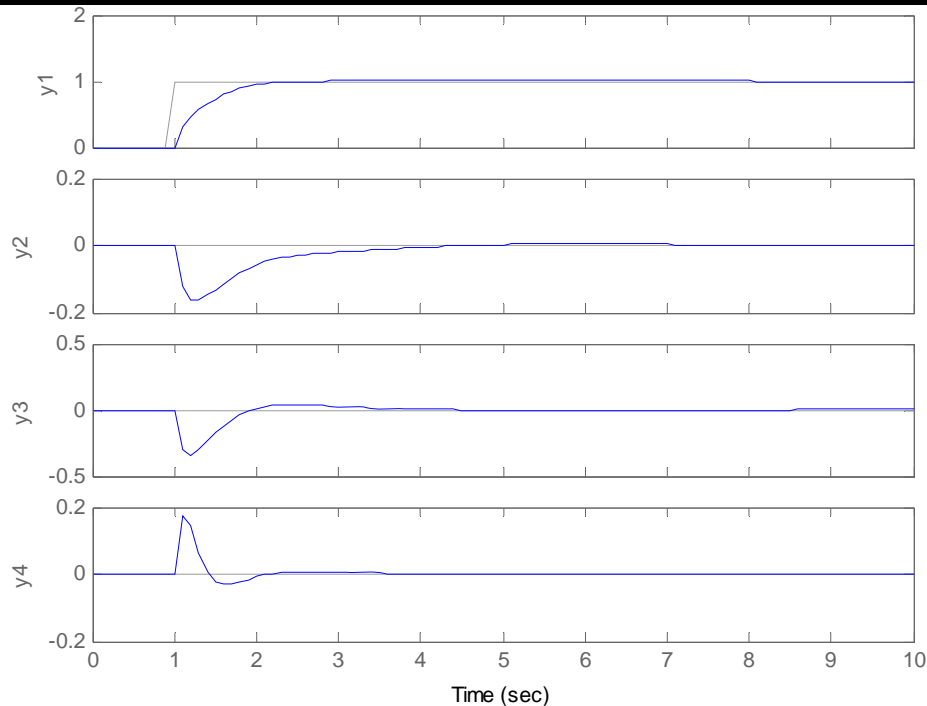


Figure 3 MPC controller performance using default parameter

#### IV. Conclusion

MPC controller offers better control performances than PI/PID controller, especially in multivariable processes. Application of MPC controller in the Alatiqi-Luyben of a 4x4 process produces the fantastic performance. To achieve an optimum performance of MPC controller, non-adaptive DMC controller tuning can be used. This method has proved that the tuning setting has the optimum performance. But, DMC controller tuning strategy applied in the Doukas-Luyben process presents the poor performance because of the DMC parameters (P and M) are too large. Nevertheless, the default setting shows the enough good performance.

#### V. Reference

- Clarke, D. W. (1996). "Adaptive Predictive Control". *A Rev. Control*. Vol. 20, pp. 83-94.
- Cutler, C. R. & D. L. Ramaker. (1980). "Dynamic matrix control—a computer control algorithm". *Proceedings of the JACC 1980*. San Francisco.
- Dougherty, Danielle and Doug Cooper. (2003). "A Practical Multiple Model Adaptive Strategy for Multivariable Model Predictive Control". *Control Engineering Practice*. (11): 649 – 664.
- Huang et. al. (2003). "A direct method for multi-loop PI/PID controller design". *Journal of Process Control* (13): 769–786
- Luyben, W.L. (1986). Simple method for tuning SISO controllers in multivariable systems. *Ind. Eng. Chem. Process Des. Dev.* 25: 654–660
- Marlin, T. (2000). *Process Control: Designing Processes and Control Systems for Dynamic Performance*. 2nd Edition, McGraw-Hill, New York.
- Monica, Thomas J., Cheng-Ching Yu, and William L. Luyben. (1988). "Improved Multiloop Single-Input/Single-Output (SISO) Controllers for Multivariable Processes". *Reprinted from I&EC RESEARCH*, 27, 969.
- Ogunnaike, et al. (1983). "Advanced Multivariable Control of a Pilot-Plant Distillation Column". *AIChE Journal* (29/ 4): 632-640.
- Qin, S.J. and T.A. Badgwell, (2003). "A survey of industrial model predictive control technology". *Control Engineering Practice*. (11): 733 – 764

