Introduction to Wavelet-Based Image Compression

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Outline

- Introduction to wavelet and wavelet transform
- Image compression scheme
- EZW – A wavelet-based image compression algorithm
- Overview of JPEG 2000
- Conclusion
- Reference

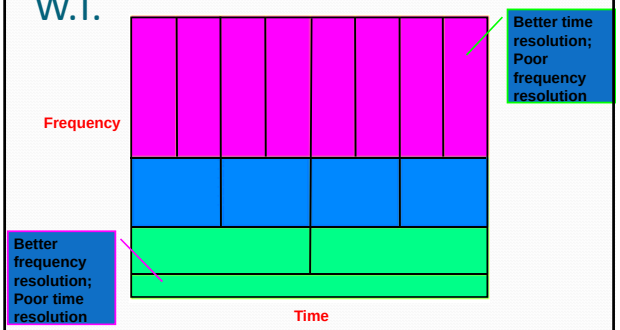
Wavelet and Wavelet Transform

- Wavelet
  - Small wave
- Wavelet transform $H(a,b)=\frac{1}{\sqrt{|b|}}\int_{-\infty}^{\infty} g(x)\psi\left(\frac{x-a}{b}\right)dx$
  - A way to decompose signal (just like Fourier transform)
  - A time-frequency analysis approach
  - Suitable for non-stationary signal
  - Notice gross features with large window
  - Notice small features with small window

Why Do We Use W.T.

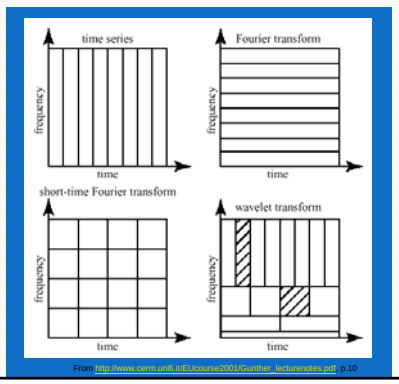
- Fourier transform?
  - Only give what frequency components exist in the signal
  - No information about time
- STFT?
  - Unchanged window
  - Dilemma of resolution
    - Narrow window -> poor frequency resolution
    - Wide window -> poor time resolution
  - Uncertainty principle

Time-Frequency Resolution of W.T.



- Each box represents a equal portion
- Resolution in STFT is selected once for entire analysis

Comparison



From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf p.10

Multi-Resolution Analysis (MRA)

- Wavelet transform
  - An alternative approach to the short time Fourier transform to overcome the resolution problem
  - Similar to STFT: signal is multiplied with a function
- Multi-resolution analysis
  - Analyze the signal at different frequencies with different resolutions
  - Good time resolution and poor frequency resolution at high frequencies
  - Good frequency resolution and poor time resolution at low frequencies
  - More suitable for short duration of higher frequency; and longer duration of lower frequency components

Multi-Resolution Analysis(MRA)

- Scaling function : $\phi(x)$

  Define : $\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k)$

  $V_j = span_k\{\phi_{j,k}(x)\}$

  Therefore

  $V_0 \subset V_1 \subset V_2 \subset \cdots \subset V_\infty$ since $\phi(x) = \sum_n h_\phi(n)\sqrt{2}\phi(2x \quad n)$
- Wavelet function : $\psi(x)$ span *difference* between adjacent scale

  Define : $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$

  $W_j = span_k\{\psi_{j,k}(x)\}$

  Therefore

  $W_j \subset V_{j+1}$ since $\psi(x) = \sum_n h_\psi(n)\sqrt{2}\phi(2x - n)$

Multi-Resolution Analysis(MRA)

- Obtain $V_{j+1} = V_j \oplus W_j$
- Recursively :

$$V_{j+1} = V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus \cdots \oplus W_j$$

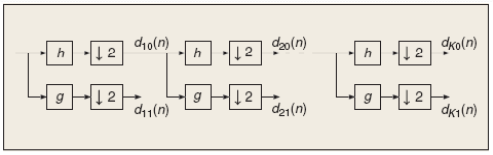
Any function $f(x)$ can be expanded as

$$f(x) = \sum_k c_{j_0}(k)\phi_{j_0,k}(x) + \sum_{j=j_0}^{\infty}\sum_k d_j(k)\psi_{j,k}(x)$$

Approximation Detail

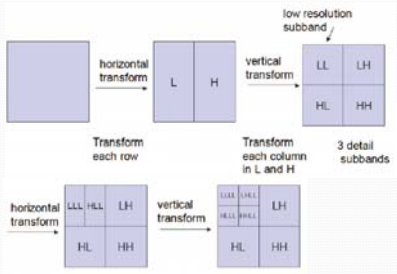
Structure of Wavelet Transform

- Analysis is just "*filtered*" and "*down-sampled*"



▲ *1. A K-level, 1-D wavelet decomposition. The coefficient notation $d_{ij}(n)$ refers to the jth frequency band (0 for low and 1 for high) of the ith level of the decomposition.*

Example of 2-D W.T.

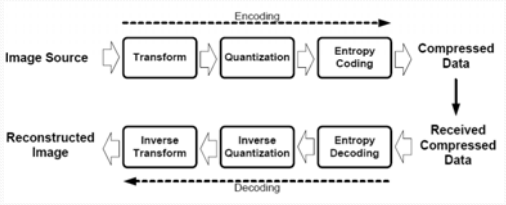


Example of 2-D W.T.

▲ *2. Original image used for demonstrating the 2-D wavelet transform.*

▲ *5. A three-level (K = 3), 2-D wavelet transform using the symmetric wavelet transform with the 9/7 Daubechies coefficients (the high-frequency bands have been enhanced to show detail).*

General Image Compression Scheme



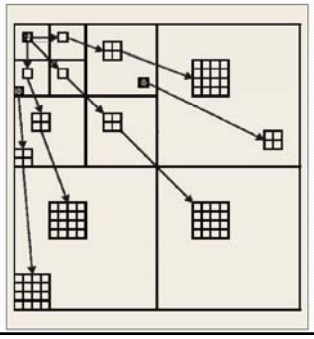
General Image Compression Scheme

- Transform
  - Decorrelate spatial signal
- Quantization
  - Drop information based on HVS
- Entropy coding
  - Encode symbols into bit-stream

Embedded Zerotree Wavelet (EZW) Coder

- A quantization and coding strategy
- Incorporates characteristics of wavelet decomposition
- Outperform some generic approach
- Fundamental concept of other wavelet-based coder
- Can be decomposed into two parts:
  - Significant map coding using zerotree
  - Successive approximation quantization

Significant Map Coding Using Zerotree



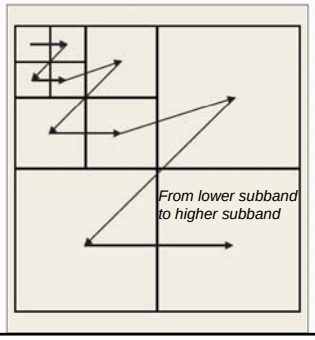
Four types of Label
1.Positive significant
2.Negative significant
3.Isolated zero
4.Zero tree root

For each coefficient:
Give a label based on predefine threshold T

$$T_0 = 2^{\lfloor \log_2 x_{max} \rfloor}$$

Significant Map Coding Using Zerotree

- Scan order :



Successive Approximation Quantization

- A refinement process
- Multi-pass scanning of coefficient using successive decreasing threshold

EZW Example (1/2)

T0 = 32

53	−22	21	−9	−1	8	−7	6
14	−12	13	−11	−1	0	2	−3
15	−8	9	7	2	−3	1	−2
34	−2	−6	10	6	−4	4	−5
−6	5	−1	1	1	3	−1	5
6	1	3	0	−2	2	6	0
4	2	1	−4	−1	0	−1	4
0	−2	7	5	−3	2	−2	3

▲ 17. An example three-level wavelet decomposition used to demonstrate the EZW algorithm.

*	−22	21	−9	−1	8	−7	6
14	−12	13	−11	−1	0	2	−3
15	−8	9	7	2	−3	1	−2
*	−2	−6	10	6	−4	4	−5
−6	5	−1	1	1	3	−1	5
6	1	3	0	−2	2	6	0
4	2	1	−4	−1	0	−1	4
0	−2	7	5	−3	2	−2	3

▲ 18. The example wavelet transform after the first dominant pass. The symbol * is used to represent symbols found to be significant on a previous pass.

EZW Example (2/2)

T0 = 32

Table 3. Resulting Output of the First Dominant Pass (T_0 = 32).

Subband	Coefficient Value	Symbol	Reconstruction Value	Comment (See Text)
LL_3	53	*p*	48	1)
HL_3	−22	*ztr*	0	2)
LH_3	14	*iz*	0	3)
HH_3	−12	*ztr*	0	
LH_2	15	*ztr*	0	
LH_2	−8	*ztr*	0	
LH_2	34	*p*	48	
LH_2	−2	*ztr*	0	
LH_1	4	*iz*	0	
LH_1	2	*iz*	0	
LH_1	0	*iz*	0	
LH_1	−2	*iz*	0	

Table 4. Resulting Output of the Subordinate Pass.

Coefficient Magnitude	Symbol	Reconstruction Magnitude
53	1	56
34	0	40

After this two step, we finish one iteration.

Ti = Ti/2(reduce the threshold)

Repeat utill target fidelity or bit-rate is achieve

Why Another Still Image Coding Standard?

- JPEG cannot fulfill the advanced requirements of today
  - Better quality and compression efficiency
  - New demands such as scalability and interoperability
- New application area imposes some new requirements.

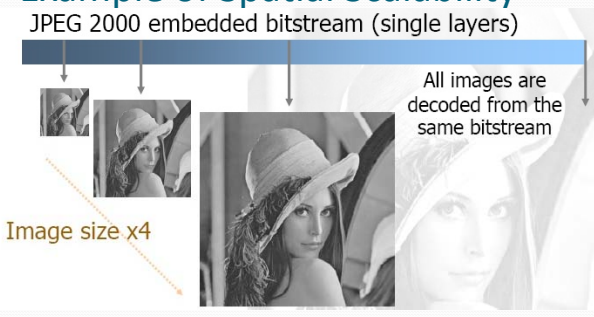
Features of JPEG2000 (1/2)

- Superior low bit-rate performance
  - Network image transmission
- Continuous-tone and bi-level compression
  - Compound documents with images and text
- Lossless and lossy compression
  - Medical images
- Progressive transmission
  - Web browsing

Features of JPEG2000 (2/2)

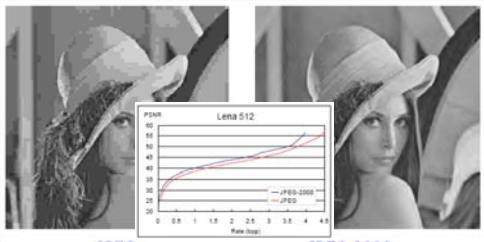
- Region-of-interest (ROI) coding
- Open architecture
  - Allow to optimize the system
- Robustness to bit errors
  - Transmission over wireless communication channel
- Protective image security
  - Watermarking, encryption ...etc

Example of Spatial Scalability



Example of ROI

Subjective Quality (0.1bpp)



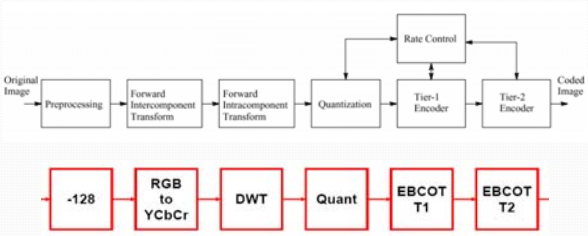
JPEG
PSNR 27.65 db

JPEG 2000
PSNR 30.86 db

Subjective Quality

@ low bit rate

JPEG2000 Compression Engine



JPEG2000 Compression Engine

- The whole compression engine can be decomposed into three part:
  - Preprocessing
  - Core processing
  - Bit-stream formation – Not included in this talk

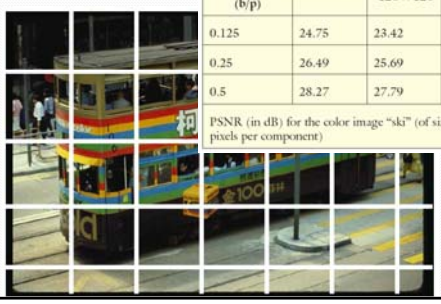
Preprocessing

- Image tiling

Table 1. The Effect of Tiling on Image Quality.

Tiling / Bit Rate (b/p)	No Tiling	Tiles of Size 128 × 128	Tiles of Size 64 × 64
0.125	24.75	23.42	20.07
0.25	26.49	25.69	23.95
0.5	28.27	27.79	26.80

PSNR (in dB) for the color image "ski" (of size 720 × 576 pixels per component)



Preprocessing

- DC level shift
  - Subtract each pixel value by 128 (2^(p-1))
- Component (Color) transformation
  - Can be lossy or lossless

$$\begin{pmatrix} Y \\ C_b \\ C_r \end{pmatrix} = \begin{pmatrix} 0299 & 0587 & 0114 \\ -016875 & -033126 & 05 \\ 05 & -041869 & -008131 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 10 & 0 & 1402 \\ 10 & -034413 & -071414 \\ 10 & 1772 & 0 \end{pmatrix} \begin{pmatrix} Y \\ C_b \\ C_r \end{pmatrix}$$

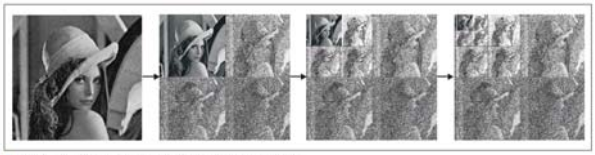
$$\begin{pmatrix} Y_r \\ V_r \\ U_r \end{pmatrix} = \begin{pmatrix} \left\lfloor \frac{R+2G+B}{4} \right\rfloor \\ R-G \\ B-G \end{pmatrix}$$

$$\begin{pmatrix} G \\ R \\ B \end{pmatrix} = \begin{pmatrix} Y_r - \left\lfloor \frac{U_r+V_r}{4} \right\rfloor \\ V_r+G \\ U_r+G \end{pmatrix}$$

Without/With Color Transform

Core Processing

- Wavelet transform
  - Can be reversible(lossless) or irreversible(lossy) according to applications
  - The standard use separable 1-D DWT for implementation



▲ 6. Three-level dyadic wavelet decomposition of the image "Lena."

Core Processing

- Filter coefficient

Table 3. Daubechies 9/7 Analysis and Synthesis Filter Coefficients.

Analysis Filter Coefficients		
i	Low-Pass Filter $h_b(i)$	High-Pass Filter $h_{bg}(i)$
0	0.6029490182363579	1.115087052456994
±1	0.2668641184428723	−0.5912717631142470
±2	−0.0782232665289785	−0.0575435262285957
±3	−0.0168641184428723	0.0912717631142470
±4	0.0267487574108976	
Synthesis Filter Coefficients		
i	Low-Pass Filter $g_b(i)$	High-Pass Filter $g_{bg}(i)$
0	1.115087052456994	0.6029490182363579
±1	0.5912717631142470	−0.2668641184428723
±2	−0.0575435262285957	−0.0782232665289785
±3	−0.0912717631142470	0.0168641184428723
±4		0.0267487574108976

Table 4. Le Gall 5/3 Analysis and Synthesis Filter Coefficients.

	Analysis Filter Coefficients		Synthesis Filter Coefficients	
i	Low-Pass Filter $h_b(i)$	High-Pass Filter $h_{bg}(i)$	Low-Pass Filter $g_b(i)$	High-Pass Filter $g_{bg}(i)$
0	6/8	1	1	6/8
±1	2/8	−1/2	1/2	−2/8
±2	−1/8			−1/8

Core Processing

- Quantization
  - Scalar quantization

$$q_b(u,v) = \text{sign}(a_b(u,v)) \left\lfloor \frac{|a_b(u,v)|}{\Delta_b} \right\rfloor$$

- Entropy coding
  - EBCOT(Embedded Block Coding with Optimal Truncation)
  - A kind of arithmetic code
  - Descendant of EZW

Conclusion

- Wavelet analysis is powerful for application which we concern different extent of detail
- Image compression is one of the major applications utilizing wavelet transform
- EZW algorithm contains fundamental idea of other wavelet-based coder
- JPEG 2000 is a new standard providing a wide range of functionality utilizing wavelet transform, which is superior to other still image coding standard

Thank You!

Reference

[1] K. Sayood, *Introduction to Data Compression. San Mateo, CA: Morgan* Kaufmann, 2000.

[2] A. Skodras, C. Christopoulos, and T. Ebrahimi, "*The JPEG2000 still image compression standard*," *IEEE Signal Processing Mag., vol. 18, pp. 36-58*, Sept. 2001.

[3] B. E. Usevitch, "*A Tutorial on Modern Lossy Wavelet Image Compression: Foundations of JPEG 2000*", *IEEE Signal Processing Magazine*, vol. 18, pp. 22-35, Sept. 2001

[4] *Advance Video Coding* Lecture Note

[5] R. C. Gonzolez, R. E. Woods, "*Digital Image Processing second edition*", Prentice Hall, 2002

[6] Wikipedia *http://en.wikipedia.org/wiki/Wiki*