Kuliah ke – 4
Program S1 Reguler DTE FTUI
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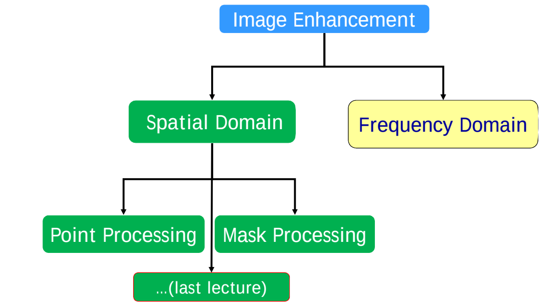
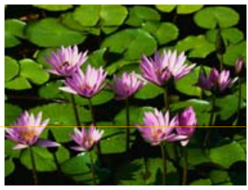
Image Enhancement (Frequency Domain)

Our topics



Frequency Concept



Line Profile: high dynamic → High Frequency Dominant

Line Profile: smooth → Low Frequency Dominant

Fourier Idea (1807)

- Fourier transform pair

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux}\, dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux}\, du.$$

- For 2-D

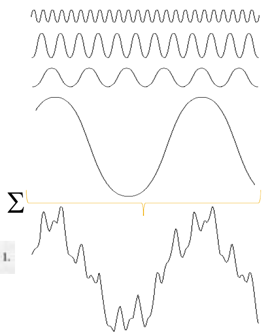
$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)}\, du\, dv.$$

- DFT of $f(x)$ for x=0,1,2,… M-1

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad \text{for } u = 0, 1, 2, \ldots, M-1.$$

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad \text{for } x = 0, 1, 2, \ldots, M-1.$$

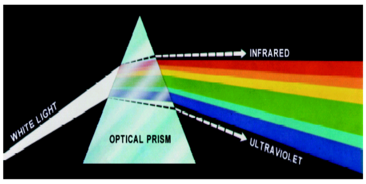


Analogy — Optical prism

Glass prism: a physical device that separates light into various color components , each depending on its wavelength (or frequency) content.

The Fourier is a "mathematical prism" that separates a function into various components, also based on frequency content.

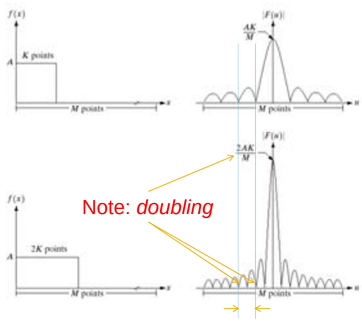
This is a powerful concept that lies at the heart of linear filtering



Basic concept — Fourier Spectrum

Signal (1-D): K(=8) points of discrete function of total M (=1024) points and its Fourier spectrum (A=1)

Result of transformation by **doubling** the number of nonzero points (into 18 points)



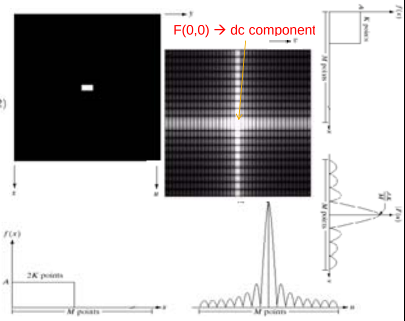
Example

Sample image:
Size: 512 × 512
Rectangle size: 20 × 40
Shifted by $(-1)^{(x+y)}$

$\Im[f(x,y)(-1)^{x+y}] = F(u - M/2, v - N/2)$

Fourier spectrum
Log transformed
$s = c\text{Log}(1+\gamma)$
$c = 0.5$

Compare to previous figure



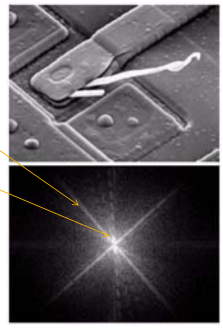
Illustration

Relation of spatial and frequency domains

SEM Image of a damaged IC (magnified 2500×) and its Fourier spectrum.

Spatial
- Strong edges 45°
- Two white oxides

Frequency
- Strong spectrum at 45°
- Slight off-axis spectrum

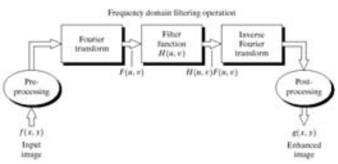


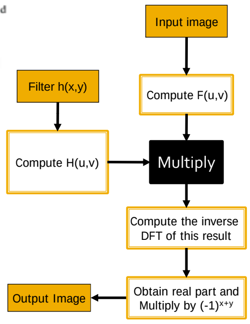
Basic Steps

1. Multiply the input image by $(-1)^{x+y}$ to center the transform, as indicated in Eq. (4.2-21).

$$\Im[f(x,y)(-1)^{x+y}] = F(u - M/2, v - N/2)$$

2. Compute $F(u, v)$, the DFT of the image from (1).
3. Multiply $F(u, v)$ by a *filter* function $H(u, v)$.
4. Compute the inverse DFT of the result in (3).
5. Obtain the real part of the result in (4).
6. Multiply the result in (5) by $(-1)^{x+y}$.





Low Pass & High Pass Filters

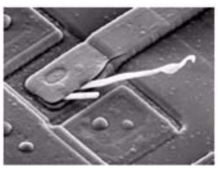
- Low frequencies in the Fourier transform are responsible for the general gray-level appearance of an image over smooth areas, while high frequencies are responsible for detail, such as edges and noise.
- A filter that attenuates high frequencies while "passing" low frequencies is called a lowpass filter. A filter that has the opposite characteristic is appropriately called a highpass filter.
- A lowpass-filtered image has less sharp detail than the original because the high frequencies have been attenuated.
- Similarly, a highpass-filtered image would have less gray level variations in smooth areas and emphasized transitional (e.g., edge) gray-leveldetail. Such an image will appear sharper.

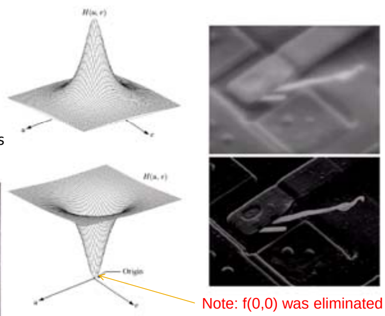
LPF and HPF

Example

Top: A two-dimensional LPF and the result of lowpass filtering the image in previous figure (below).

Bottom: A two-dimensional HPF and the result of highpass filtering the same image

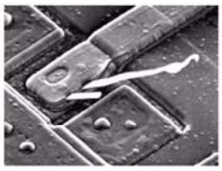


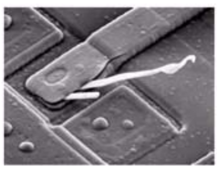


Filter height

Filter modification

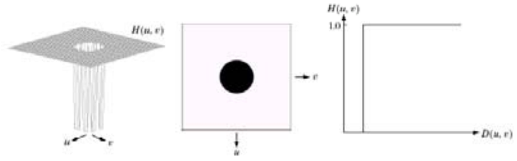
Result of highpass filtering the image with the previous HPF, but modified by adding a constant of 1.5 the filter height to the filter function.





Ideal High Pass Filter

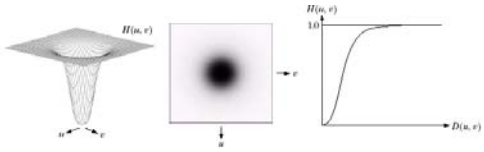
- H(u,v) = 0 if $D(u,v) \le D_0$
  = 1 if $D(u,v) > D_0$



Butterworth HPF

$$H(u,v) = \frac{1}{1+[D_0 / D(u,v)]^{2n}}$$

$n = order$



Gaussian LPF & HPF

Freq. Domain

Spatial Domain

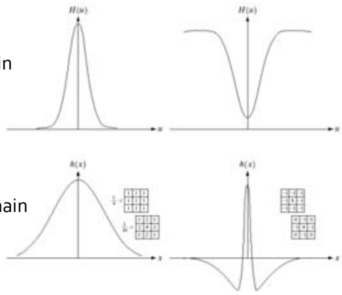


FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

Ideal Low Pass Filter (ILPF)

- H(u,v) = 1 if $D(u,v) \le D_0$
  = 0 if $D(u,v) > D_0$
- D_0 : cutoff frequency locus, $D_0 > 0$
- D(u,v) : the distance of (u,v) to the origin.
- $D(u,v) = (u^2+v^2)^{1/2}$

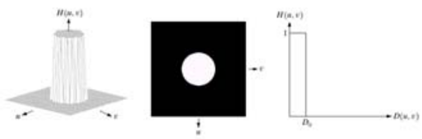
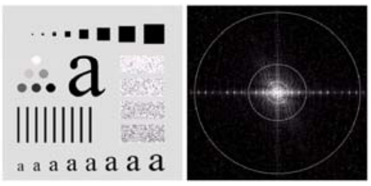


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Freq. Domain LPF, HPF

Example

Left: An image size 500×500
Right: Fourier spectrum
Superimposed circles have radii values of: 5, 15, 30, 80 and 230 which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively



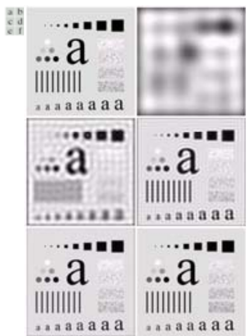
Ideal Lowpass Filters

Filtering Results

(a) Original Image

Result of ILPF with cutoff freq. set at radii values of :

(b) 5, power removed 8.0%
(c) 15 , power removed 5.4%
(d) 30 , power removed 3.6%
(e) 80 , power removed 2.0%
(f) 230 , power removed 0.5%



Filtering as convolution

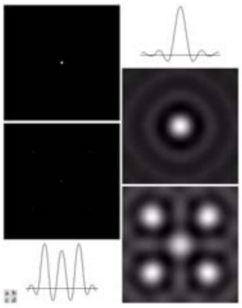
Ringing

(a) A frequency-domain ILPF of radius 5

(b) Corresponding spatial filter (note the ringing)

(c) Five impulses in the spatial domain, simulating the values of five pixels

(d) Convolution of (b) and (c) in the spatial domain



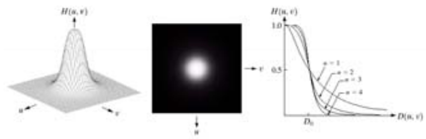
Butterworth Low Pass Filter

BLPF

(a) Perspective plot of Butterworth lowpass filter transfer function

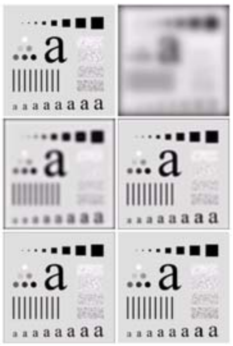
(b) Filter displayed as an image

(c) Filter radial cross section of orders 1 through 4

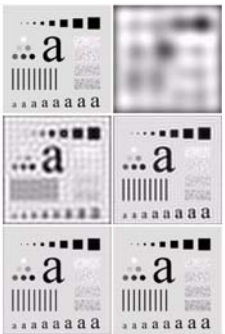


$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}} \qquad n = order$$

BLPF Results



Compare to ILPF →



Spatial Representations

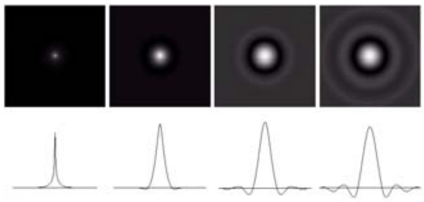


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Gaussian LPF

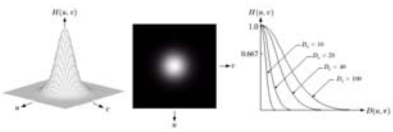
Profile

a. Perspective plot of a GLPF transfer function

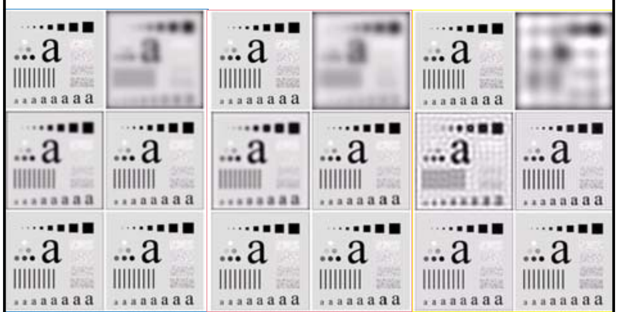
b. Filter displayed as an image

c. Filter radial cross sections for various values of D_0

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$



GLPF compared to other LPFs

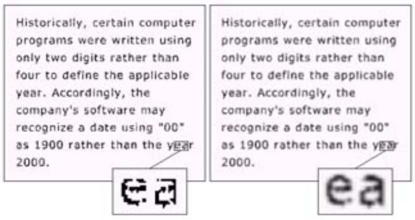


GLPF BLPF ILPF

Sample — Application

Sample text of poor resolution (broken characters in magnified view)

Result of filtering with a GLPF (broken character segments were joined)



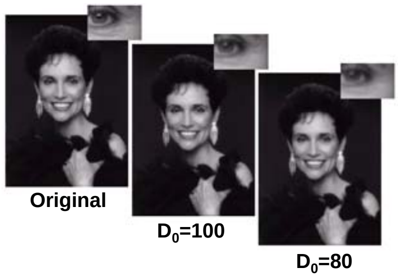
GLPF Application — Another Example

Original image (1024 × 732)

Result of filtering with a **GLPF (D_0=100)**

Result of filtering with a **GLPF (D_0=80)**

Note reduction in skin **fine lines** in the magnified sections of each filtering result



GLPF for Remote Sensing Data — Another Example

Prominent scan lines in remote sensing imagery

Result of using a **GLPF (D_0=30)**

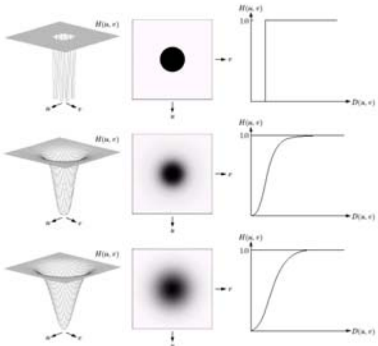
Result of using a **GLPF (D_0=10)**

Highpass Filters — High Pass Filters: Ideal (IHPF), Butterworth (BHPF), Gaussian (GHPF)

Top: Perspective plot, image representation, and cross section of a typical **Ideal HPF**

Middle: the same sequence for typical **Butterworth HPF**

Bottom: the same sequence for typical **Gaussian HPF**

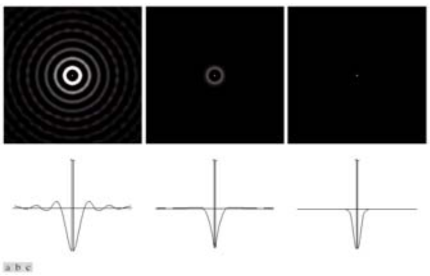


Filter Representation — Spatial Representations

Spatial representation of typical:

(a) Ideal
(b) Butterworth
(c) Gaussian

Frequency domain HPF and corresponding gray-level profiles

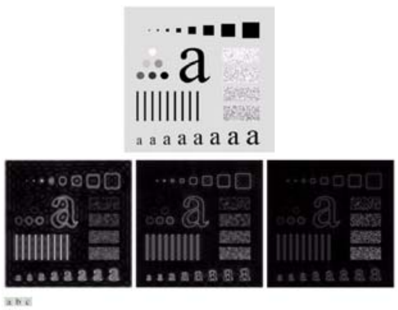


Highpass filtering — IHPF Results

Top: Original image

Bottom: Results of ideal highpass filtering with:
(a) D_0=15
(b) D_0=30
(c) D_0=80

Note: problems of ringing are quite evident in (a) and (b)



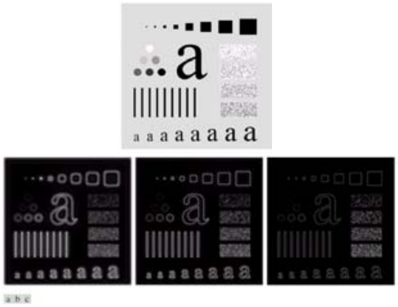
Highpass filtering

BHPF Results

Top: Original image

Bottom: Results of Butterworth highpass filtering with:
(a) Order= 2; D_0=15
(b) Order= 2; D_0=30
(c) Order= 2; D_0=80

Note: these results are much smoother than those obtained with an IHPF



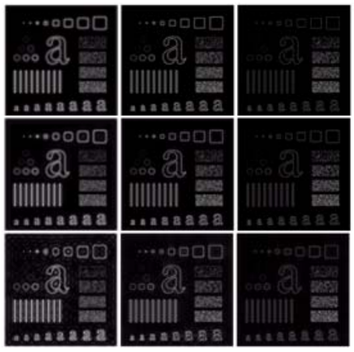
Gaussian HPF

GHPF Results

(1) Results of Gaussian highpass filtering with:
(a) Order= 2; D_0=15
(b) Order= 2; D_0=30
(c) Order= 2; D_0=80

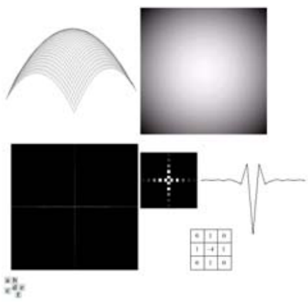
Compare to:
(2) BHPF
(3) IHPF

(1)
(2)
(3)



Laplacian Filter

(a) 3-D plot of Laplacian in the freq. domain
(b) Image representation of (a)
(c) Laplacian in the spatial domain from the inverse DFT of (b)
(d) Zoomed section of the origin of (c)
(e) Gray-level profile through the center of (d)
(f) Laplacian mask

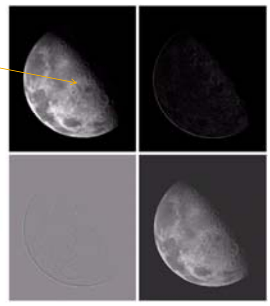


Laplacian Filter

Laplacian Filtering Result

(a) Image of the North Pole of the moon
(b) Laplacian filtered image
(c) Laplacian image scaled
(d) Image enhanced using

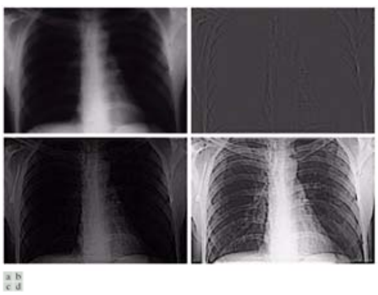
$$g(x, y) = f(x, y) - \nabla^2 f(x, y).$$



Combined processing

Application

(a) A chest X-ray image
(b) Result of Butterworth highpass filtering
(c) Result of high-frequency emphasis filtering
(d) Result of histogram equalization of (c)

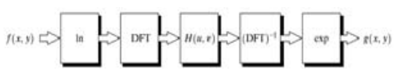


Another approach

Homomorphic Filter

Top: homomorphic filtering approach for image enhancement

Bottom: Cross section of a circularly symmetric filter function. D(u,v) is the distance from the origin of the centered transform



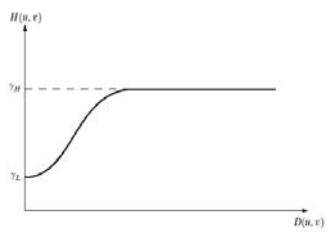
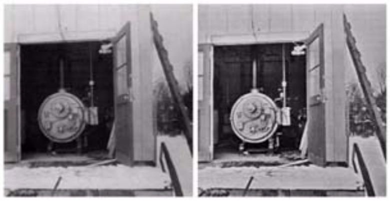


Image detailing

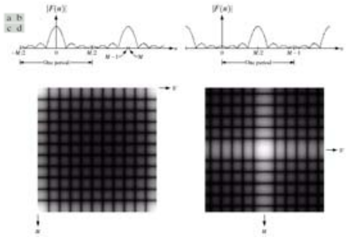
Example

(a) Original image
(b) Image processed by homomorphic filtering

Note: details inside shelter

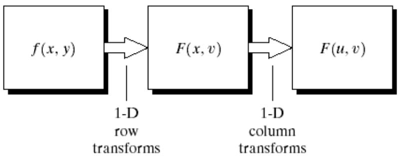


Back-to-Back



(a) Fourier spectrum showing back-to-back half periods in the interval [0,M-1]
(b) Shifted spectrum showing a full period in the same interval
(c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions
(d) Centered Fourier spectrum

Computation 2-D FT



Computation of the 2-D Fourier transform as a series of 1-D transforms

FT Important Properties

Property	Expression(s)
Fourier transform	$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(ux/M+vy/N)}$
Inverse Fourier transform	$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$
Polar representation	$F(u,v)=\lvert F(u,v)\rvert e^{-j\phi(u,v)}$
Spectrum	$\lvert F(u,v)\rvert=\left[R^2(u,v)+I^2(u,v)\right]^{1/2}$, $R=\text{Real}(F)$ and $I=\text{Imag}(F)$
Phase angle	$\phi(u,v)=\tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$
Power spectrum	$P(u,v)=\lvert F(u,v)\rvert^2$
Average value	$\bar{f}(x,y)=F(0,0)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)$
Translation	$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)}\Leftrightarrow F(u-u_0,v-v_0)$ $f(x-x_0,y-y_0)\Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M+vy_0/N)}$ When $x_0=u_0=M/2$ and $y_0=v_0=N/2$, then $f(x,y)(-1)^{x+y}\Leftrightarrow F(u-M/2,v-N/2)$ $f(x-M/2,y-N/2)\Leftrightarrow F(u,v)(-1)^{u+v}$

FT Important Properties

Conjugate symmetry	$F(u,v)=F^*(-u,-v)$ $\lvert F(u,v)\rvert=\lvert F(-u,-v)\rvert$
Differentiation	$\frac{\partial^n f(x,y)}{\partial x^n}\Leftrightarrow (ju)^n F(u,v)$ $(-jx)^n f(x,y)\Leftrightarrow \frac{\partial^n F(u,v)}{\partial u^n}$
Laplacian	$\nabla^2 f(x,y)\Leftrightarrow -(u^2+v^2)F(u,v)$
Distributivity	$\Im[f_1(x,y)+f_2(x,y)]=\Im[f_1(x,y)]+\Im[f_2(x,y)]$ $\Im[f_1(x,y)\cdot f_2(x,y)]\neq\Im[f_1(x,y)]\cdot\Im[f_2(x,y)]$
Scaling	$af(x,y)\Leftrightarrow aF(u,v)$, $f(ax,by)\Leftrightarrow\frac{1}{\lvert ab\rvert}F(u/a,v/b)$
Rotation	$x=r\cos\theta \quad y=r\sin\theta \quad u=\omega\cos\varphi \quad v=\omega\sin\varphi$ $f(r,\theta+\theta_0)\Leftrightarrow F(\omega,\varphi+\theta_0)$
Periodicity	$F(u,v)=F(u+M,v)=F(u,v+N)=F(u+M,v+N)$ $f(x,y)=f(x+M,y)=f(x,y+N)=f(x+M,y+N)$
Separability	See Eqs. (4.6-14) and (4.6-15). Separability implies that we can compute the 2-D transform of an image by first computing 1-D transforms along each row of the image, and then computing a 1-D transform along each column of this intermediate result. The reverse, columns and then rows, yields the same result.

FT Important Properties

Property	Expression(s)
Computation of the inverse Fourier transform using a forward transform algorithm	$\frac{1}{MN}f^*(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F^*(u,v)e^{-j2\pi(ux/M+vy/N)}$ This equation indicates that inputting the function $F^*(u,v)$ into an algorithm designed to compute the forward transform (right side of the preceding equation) yields $f^*(x,y)/MN$. Taking the complex conjugate and multiplying this result by MN gives the desired inverse.
Convolution[†]	$f(x,y)\star h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$
Correlation[†]	$f(x,y)\star h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f^*(m,n)h(x+m,y+n)$
Convolution theorem[†]	$f(x,y)\star h(x,y)\Leftrightarrow F(u,v)H(u,v)$; $f(x,y)h(x,y)\Leftrightarrow F(u,v)\star H(u,v)$
Correlation theorem[†]	$f(x,y)\star h(x,y)\Leftrightarrow F^*(u,v)H(u,v)$; $f^*(x,y)h(x,y)\Leftrightarrow F(u,v)\star H(u,v)$

FT Important Properties

Some useful FT pairs:

Impulse	$\delta(x, y) \Leftrightarrow 1$
Gaussian	$A\sqrt{2\pi}\sigma e^{-2\pi^2\sigma^2(x^2+y^2)} \Leftrightarrow Ae^{-(u^2+v^2)/2\sigma^2}$
Rectangle	$\text{rect}[a, b] \Leftrightarrow ab\frac{\sin(\pi ua)}{(\pi ua)}\frac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow \frac{1}{2}\left[\delta(u + u_0, v + v_0) + \delta(u - u_0, v - v_0)\right]$
Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow j\frac{1}{2}\left[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)\right]$

[†] Assumes that functions have been extended by zero padding.

Tugas

Kerjakan menggunakan MATLAB®

- Baca sebuah citra B/W
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