Lecture Number – 10
Program S1 Reguler, DTE FTUI
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Representation & Description

Representation

Chain Code

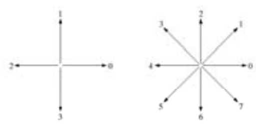
- represent a boundary by a connected sequence of straight-line segment of specified length and direction
- based on 4-or- 8 connectivity of the segments
- can be generated following a boundary in a clockwise direction and assigning a direction to the segments connecting every pair of pixels
  - is unacceptable for two principal reasons:
    - (1) too long code length
    - (2) the shape boundary can be disturbed by noise or imperfect segmentation
  - resample the boundary by selecting a larger grid spacing

Normalize with respect to the starting point

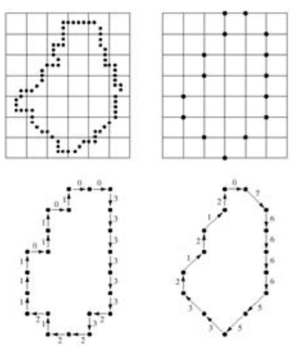
- Treat the chain code as a circular sequence of direction numbers
- Redefine the starting point
- Normalize for rotation by using the first difference of the chain code (counting the number of direction changes)

Direction Numbers

a b
FIGURE 11.1 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.



Digital Boundary



a b
c d
FIGURE 11.2 (a) Digital boundary with resampling grid superimposed. (b) Result of resampling. (c) 4-directional chain code. (d) 8-directional chain code.

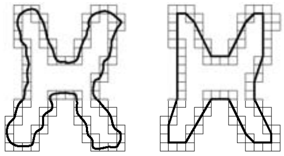
Polygonal approximations

- The goal: capture the “essence” of the boundary shape with the fewest possible polygonal segments
- Minimum perimeter polygons
  - Find minimum perimeter polygons: shrink
  - If each cell encompasses only one point---the error in each cell would be $\sqrt{2d}$
- Merging techniques
  - Merge points along a boundary until the least square error line fit of the points merged exceeds a preset threshold
  - When the above condition occurs, repeat the procedure
  - At the end: the intersection of adjacent line segments form the vertices of the polygon

Splitting techniques

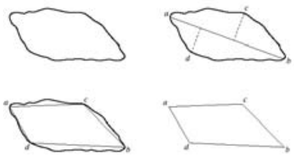
- Criterion for splitting: the maximum perpendicular distance from a boundary segment to the line joining its end points not exceed a preset threshold
  - A close boundary: the best starting points are the two farthest points

Object Boundary



a b
FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.

Boundary to Polygon



a b
c d
FIGURE 11.4 (a) Original boundary. (b) Boundary divided into segments based on extreme points. (c) Joining of vertices. (d) Resulting polygon.

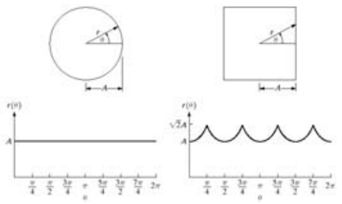
Signatures

- a 1-D functional representation of a boundary and may be generated in
  - plot the distance from the centroid to the boundary as a function of angle
- reduce the boundary representation from 2-D to a 1-D function
- are invariant to translation, but depend on rotation and scaling
  - solution to rotation invariant
    - normalize with respect to rotation can be achieved by selecting the same starting point, regardless of the shapes orientation
    - select the point on the eigen axis that is farthest from the centroid
    - obtain the chain code, and assume that the coding is coarse enough so that rotation does not affect its circularity

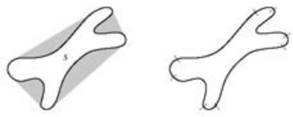
Solution to scaling

- based on uniformity in scaling with respect to both axes and that sampling is taken at equal interval of θ (changes in size of a shape results in changes in the amplitude value of the corresponding signature)
  - scale all functions so that they always span the same range of values [0,1]
- The advantage of this method: simplicity
- The disadvantage of this method: depending on maximum and minimum values (especially in noisy shape: dependence from object to object)
  - divide each sample by the variances of the signature
  - remove the dependence on size while preserving the fundamental shape of the waveform

- another way to generate signature
  - transverse the boundary and, corresponding to each point on the boundary, plot the angle between a line tangent to the boundary at that point and a reference line
  - would carry information about basic shape characteristics
- slope density function as a signature
  - is a histogram of tangent-angle values
  - correspond to sections of the boundary with constant tangent angles



a b
FIGURE 11.5 Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \le \theta \le \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \le \pi/2$.



a b
FIGURE 11.6 (a) A region, S, and its convex deficiency (shaded). (b) Partitioned boundary.

Boundary segments

- decompose a boundary into segments
- reduce the boundary's complexity and simplify the description process
  - is attractive when the boundary contains one or more significant concavities
  - convex hull and convex deficiency
  - digital boundaries tend to be irregular because of digitization, noise, and variation in segmentation

Irregular digital boundary

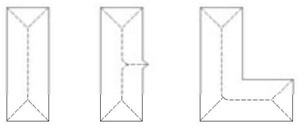
- cause convex deficiency have small, meaningless components scattered randomly throughout the boundary
- resolution: smooth a boundary prior to partitioning
  1) transverse the boundary and replace the coordinate of each pixel (for small irregularities, but it is time consuming and difficult to control)
  2) use polygon approximation prior to finding the convex deficiency

Skeletons

- reduce to a graph by a skeleton to represent a shape
- Obtain the skeleton of a region via a thinning
- morphology skeleton cannot keep the skeleton connected
- Def. of a skeleton
  - may be defined via the medial axis transformation (MAT)
  - The MAT of a region R with border B
    - For each point p in R, find its closet neighbor in border B
    - "Closet" depends on Euclidean distance
    - yield an intuitively pleasing skeleton
    - computation expensive
  - Maximum disk

- thinning algorithm iteratively delete edge points of a region subject to the three constraints:
  (1) does not remove endpoints ;
  (2) does not break connectivity;
  (3) does not cause excessive erosion of the region
- two steps applied to the contour points of the given region
  (1) flag a contour point for deletion based on four conditions (a) (b) (c) (d): If one or more of conditions (a) –(d) are violated, the value is not changed, otherwise flag for deletion; the point is not deleted until all border points have been processed

- (2) is applied to the resulting data based on (a) (b) (c′) (d′)
  - N(p1):The number of nonzero neighbors of p1
  - T(p1): the number of 0-1 transitions in the ordered sequence p2, p3,,, p8, p9
    - (a): end point
    - (b): 1 pixel thick
    - (c) (d) p4=0 (east); or p6=0 (south) or p2=0 and p8=0 "north west" corner point



a b c

FIGURE 11.7 Medial axes (dashed) of three simple regions.

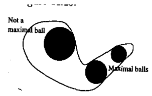


Figure 11.23: *Ball and maximal balls in Euclidean plane.*

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

FIGURE 11.8 Neighborhood arrangement used by the thinning algorithm.

FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

0	0	1
1	p_1	0
1	0	1

FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.

Boundary descriptor

Some simple boundary descriptor

- the length for a chain code curve with unit spacing
  - the diameter of a boundary B is defined as $Diam(B) = \max_{i,j}[D(p_i, p_j)]$
  - major axis: constructed form the diameter and the orientation of a line segment connecting two extreme points
  - minor axis: line perpendicular major axis
  - basic rectangle: the box passing through the outer four points of intersection of the boundary
  - eccentricity: the ratio of the major to the minor axis
- curvature: the rate of change of slope (the difference between the slope of adjacent boundary segments)

- convex segment: the change in slope at p is nonnegative
- Concave segment
- A nearly straight segment: slope change < 10^o
- Corner point: if the change exceeds 90^o

11.2.2 Shape numbers

- based on the 4-direction code
- the first difference of smallest magnitude
- the order n of a shape number is defined as the number of digits
  - n is even for a closed boundary
  - depend on the orientation of the grid
- Desired shape number - the rectangle of n whose eccentricity best approximation that of the best rectangle; For example n=12,

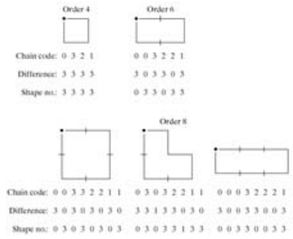
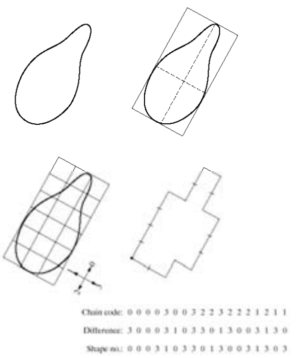


FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.



a b
c d

FIGURE 11.12 Steps in the generation of a shape number.

Fourier descriptor

- represent a boundary as the sequence of coordinates S(k)=[x(k),y(k)]
- each coordinate pair can be treated as complex number so that s(k)=x(k)+jy(k)
- reduce a 2-D to 1-D problem
- the Fourier descriptor of the boundary: the complex coefficients a(u)
- the inverse Fourier transform of these coefficients restores s(k) (11.2-4) $S(k) = \sum_{u=0}^{p-1} a(u) e^{j2\pi uk/K}$
  - only first P coefficients are used (11.2-5): $\hat{S}(k) = \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}$
- directly insensitive to geometrical changes: translation, rotation and scaling
  - translation has no effect on the descriptor except for u=0
- Results depend on the order in which points are processed

- Insensitive to starting point
- Table 11.1 summarizes the Fourier descriptors for a boundary sequence s(k)

Statistical moments

- Describe a shape quantitatively by using statistical moment
- represent a line segment as a 1-D function g® is obtained by
  - connecting the two end points of the segment
  - rotating the line segment until its is horizontal
- treat the amplitude of g as a discrete random variable v and form an amplitude histogram
  - The nth moment of v about its mean is $\mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i)$
  - The second moment measures the spread of the curve about the mean value
  - The third moment measures its symmetry

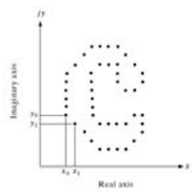


FIGURE 11.13 A digital boundary and its representation as a complex sequence. The points (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

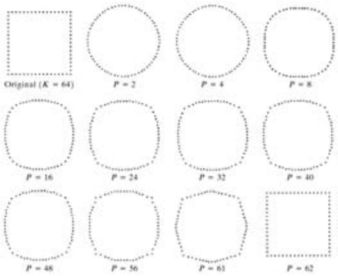


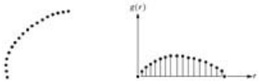
FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

TABLE 11.1 Some basic properties of Fourier descriptors.

a b

FIGURE 11.15 (a) Boundary segment. (b) Representation as a 1-D function.



Regional descriptors

Simple descriptors

- Area
- Perimeter
- Compactness
- Maximum and Minimum gray values

Topological descriptor

- Properties of a figure that are unaffected by any deformation, as long as there is no tearing r joing of the figure
- Euler number E=C-H
- Euler formula V-Q+F=C-H=E

Region no. (from top)	Ratio of lights per region to total lights
1	0.204
2	0.640
3	0.049
4	0.107

FIGURE 11.16 Infrared images of the Americas at night. (Courtesy of NOAA.)

FIGURE 11.17 A region with two holes.

FIGURE 11.18 A region with three connected components.

a b

FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

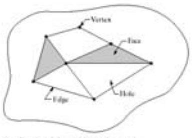
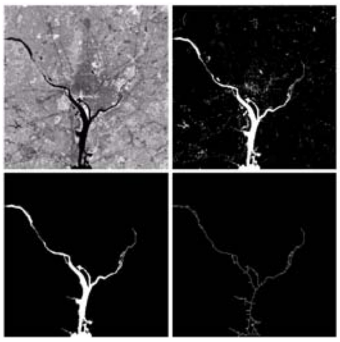


FIGURE 11.20 A region containing a polygonal network.



a b
c d

FIGURE 11.21 (a) Infrared image of the Washington, D.C. area. (b) Thresholded image. (c) The largest connected component of (b). Skeleton of (c).

Texture

- Provide measures of properties such as smoothness, coarseness, and regularity.
- Three principal approaches used in image processing: statistical, structural, and spectral.
  - statistical: yield characterization of texture as smooth, coarse, and grainy
  - structural: based on regularly spaced parallel lines
  - spectral: based on Fourier spectrum and detect global periodicity

Statistical

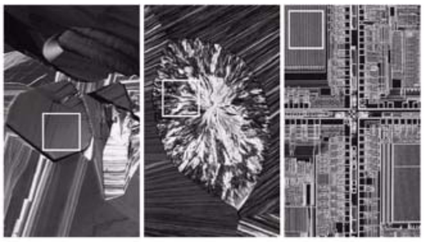
- Use statistical moments of the gray level histogram of an image or region
- The nth moment if z about the mean is:
  - The second moment is a measure of gray-level contrast that can be used to describe relative smoothness
  - The third moment,   is measure of the skewness of the histogram
  - The fourth moment is measure of its relative flatness
  - The fifth and higher moments are not so easily related to histogram shape

- Uniformity: maximally uniform
- Average entropy measure: is a measure of variability and is 0 for a constant image
- Disadvantage of measures using only histogram: suffers from the limitation that they carry no information regarding the relative position of pixels with respect to each other
  - Resol: consider not only the distribution of intensities, but also the positions of pixels with nearly equal intensities

- Let P be a position operator and Let A be a k×k matrix whose element is the number of times that points with gray level zj
  - For instance:

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix} \qquad \begin{matrix} 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{matrix}$$

- Gray-level co-occurrence matrix: is formed by dividing every element of A by n
  - A set of descriptors useful for texture based on co-occurrence
  - Maximum probabilities: $\max_{i,j}(c_{i,j})$
  - Element difference moment of order k: $\sum_i \sum_j c_{ij}/(i-j)^k$
  - Inverse element difference moment of order k: $\sum_i \sum_j (i-j)^k c_{ij}$
  - Uniformity: $\sum_i \sum_j c_{ij}^2$
  - Entropy: $-\sum_i \sum_j c_{ij} \log_2 c_{ij}$



a b c

FIGURE 11.22 The white squares mark, from left to right, smooth, course, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

TABLE 11.2 Texture measures for the subimages shown in Fig. 11.22.

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

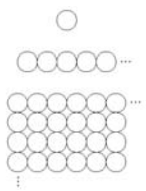
- Structural approaches
  - A rule of the form S→a, which the symbol S may be rewritten as aS
    - Scheme: S→bA, A→cA, A →c, A →bS, S →a (B: cirle down; c:circle to the left)
      - Generate a string of the form aaabccbaa that corresponds to a 3×3 matrix
  - Texture primitive can be used to form complex texture patterns by means of some rules

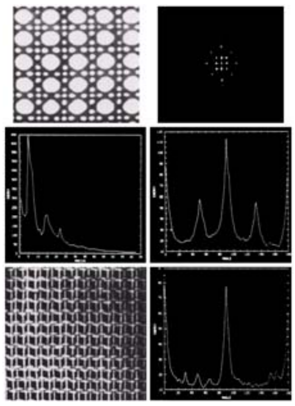
Spectral approaches

- Use Fourier spectrum to describe the directionality of periodic or almost periodic @-D patterns
- Three features of the Fourier spectrum
  - Prominent peaks: give the principal direction of the texture patterns
  - The location of the peaks in the frequency plane: give the fundamental spatial periods of the patterns
  - Eliminating any periodic components via filtering: leaves non-periodic image elements, which can then be described by statistical techniques

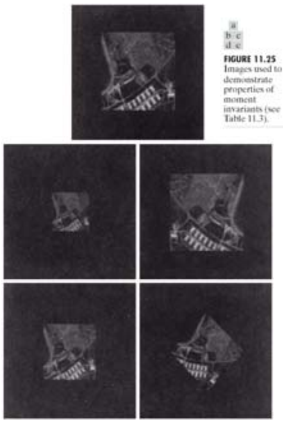
a b c

FIGURE 11.23 (a) Texture primitive. (b) Pattern generated by the rule $S \to aS$. (c) 2-D texture pattern generated by this and other rules.





a b c d e f FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)



a
b c
d e

FIGURE 11.25 Images used to demonstrate properties of moment invariants (see Table 11.3).

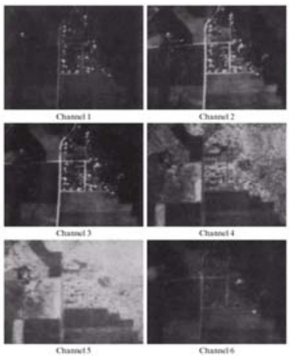
Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated 45°
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

TABLE 11.3 Moment invariants for the images in Figs. 11.25(a)-(e).

Moment of 2-D functions

- The moment of @-D continuous function
  - The central moment $\mu_{pq} = \int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q f(x,y)dxdy$
  - The central moment of order up to 3
  - The normalized central moments
    $$m_{pq} = \int_{-\infty}^{\infty} x^p x^q f(x,y)dxdy$$
  - A set of seven moments
    - Invariant to translation, rotation, and scaling
    - Error caused by digitization

FIGURE 11.26 Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



Use of principal component

- Mean vector
- covariance matrices Cx=E{(x-mx)(x-mx)T}
- Cx is real and symmetric, finding a set of n orthonormal eigenvector is possible
  - Xi and Xj is uncorrelated → Cij=Cji=0
- Use A as a transformation matrix to map x's into vectors denoted by y's as follows (Hotelling transform): y=A(x-mx)
- my=E{y}=0; Cy =ACx AT
- Reconstruct data for x=ATy+mx

TABLE 11.4 Channel numbers and wavelengths.

Channel	Wavelength band (microns)
1	0.40-0.44
2	0.62-0.66
3	0.66-0.72
4	0.80-1.00
5	1.00-1.40
6	2.00-2.60

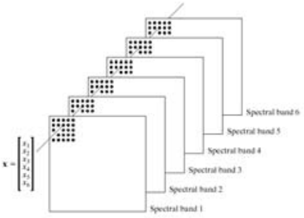


FIGURE 11.27 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

TABLE 11.5 Eigenvalues of the covariance matrix obtained from the images in Fig. 11.26.

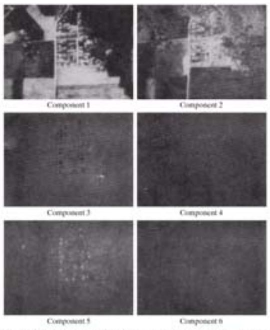
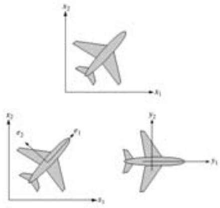


FIGURE 11.28 Six principal-component images computed from the data in Fig. 11.26. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



a
b c

FIGURE 11.29 (a) An object. (b) Eigenvectors. (c) Object rotated by using Eq. (11.4-6). The net effect is to align the object along its eigen axes.

Relational descriptors

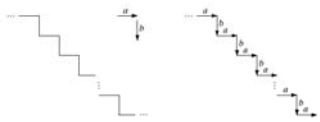
- Capture in the form of rewriting rules basic repetitive patterns in a boundary or region
  - The staircase structure
  - Formulate a recursive relationship involving these primitive elements
  - Use the rewriting rules (a) $S \rightarrow aS$; (b) $A \rightarrow bS$ and (c) $A \rightarrow b$
- Reduce 2-D positional relations to 1-D form
- Applications of strings to image description
  - Extract line segment from objects of interest
    - Follow the contour of an object and code the results with segments of specified direction and/or length
    - String descriptions are best suited for applications in which connectivity of primitives can be expressed in a head-to-tail

Tree descriptors

- Definition
  - A unique node: the root
  - The remaining nodes are partitioned into m disjoined sets T1,…, T m (subtree)
- The tree frontier: the set of nodes at the bottom of the tree (the leaves)
- Two important information

  (1) A node stored as a set of words describing the node
  - Identifies an image substructure (region or boundary segment)

  (2) A node to its neighbor, stored as a set of pointers to those neighbors
  - Defines the physical relationship of that substructure to other substructure



a b
FIGURE 11.30
(a) A simple staircase structure.
(b) Coded structure.

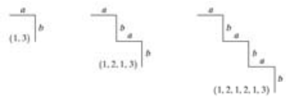
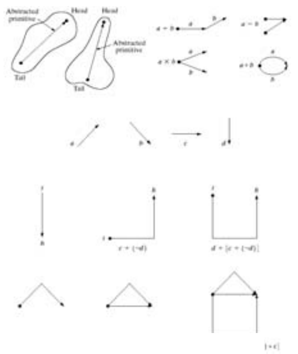


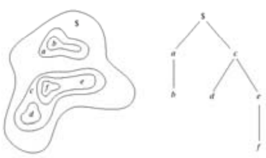
FIGURE 11.31
Sample derivations for the rules $S \to aA$, $A \to bS$, and $A \to b$.

FIGURE 11.32
Coding a region boundary with directed line segments.



a b
c
d
FIGURE 11.33 (a) Abstracted primitives. (b) Operations among primitives. (c) A set of specific primitives. (d) Steps in building a structure.

FIGURE 11.34 A simple tree with root $ and frontier *xy*.



a b
FIGURE 11.35 (a) A simple composite region. (b) Tree representation obtained by using the relationship "inside of."