Electrical Capacitance Volume Tomography static imaging using Compressive Sensing with l1 sparse recovery

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Abstract—Compressive Sensing (CS) framework is mathematical framework to recover the signal by having less measurement data compared to Shannon-Nyquist theorem. It indicates the underdetermined linear system where the dimension of measurement data is much lower compared to dimension of the projected data. The basic idea of CS is to shift the sensing load into image reconstruction load. Thus, even though the sensing process produces less measurement data subject to the recovery data dimension, the CS theoretically is able to perform good signal recovery. Theoretically, CS should be working for natural sparse signal or sparse in transform domain. Electrical Capacitance Volume Tomography (ECVT) imaging forms naturally underdetermined linear system since the dimension of capacitance as the measurement data is much lower compared to dimension of predicted permittivity distribution. In addition, the ECVT signal is naturally sparse. Thus, the compressive sensing framework is theoretically promising for ECVT imaging. The early simulations show that compressive sensing with $l_1$ optimization on the sparse recovery succeeds to eliminate the elongation error on ECVT imaging by ILBP (Iterative Learning Back Propagation).

Keywords—Compressive Sensing, ECVT imaging, $l_1$ Optimization

I. INTRODUCTION

ECVT is a tomography which utilizes capacitance measurement on the sensing boundary [1]. The aim of it is to investigate the perturbation inside the sensing domain [1]. Instead of collecting the 2D imaging and stack them up to produce 3D imaging as the previous technology did (ECT), ECVT, which utilizes the fringing effect, is able to do direct observation of 3D object in real time more precisely [2]. Thus for more rapid observation, ECVT is more powerful compared to ECT.

Along with the ECVT research and development, imaging method is one of the most concerned to support ECVT better performance besides the sensor and data acquisition technique. The soft field properties of electrical based tomography and ill-condition measurement matrix are some crucial factors which leads to the complicated inverse problem (ECVT Imaging).

Some ECVT imaging technique such as NN-MOIRT [1], Combined Feed Forward NN [3] and ILBP [1] have been published. NN-MOIRT reported performs very powerful especially for dynamic imaging. However, due to simplicity and computation issue, ILBP is still practically used.

Compressive Sensing framework is a new approach that qualify signal recovery of certain signal that is naturally sparse or sparse in certain domain by utilizing a number of linear projection which its dimension is considerably lower than the number of sample required by the Shannon-Nyquist Theorem [4, 5]. Theoretically, the CS framework meet the ECVT imaging system problem definition. In ECVT Imaging system, the number of the capacitance measurement is practically much lower compared to the dimension of the projected data. Hence it leads to the underdetermined linear system.

This paper elaborates the compressive sensing framework with $l_1$ optimization for sparse recovery for ECVT imaging for static object. The elaboration will discuss the crucial part on modelling compressive sensing for ECVT imaging and present the early simulation on ECVT imaging for static object. The early simulations showed that compressive sensing with $l_1$ optimization on the sparse recovery succeeds to eliminate the elongation error that is exist on ECVT imaging by ILBP (Iterative Learning Back Propagation).

II. ECVT (ELECTRICAL CAPACITANCE VOLUME TOMOGRAPHY)

ECVT is a technique that able to reconstruct simultaneously a volume image of a region inside the sensing domain by utilizing the capacitance measurement [1]. It is invasive since the electrodes are attached on the wall out of the vessel to measure the capacitance. Fringing effect is used to produce the volumetric imaging which differentiate it to the previous capacitive tomography technology, ECT (electrical Capacitance Tomography) [1, 2]. The volumetric imaging by ECVT makes 3D imaging for real time observation is more reliable compared to conventional 3D imaging by ECT [1].
A. The ECVT Component

The hardware of ECVT consist of three main parts: sensing system, data acquisition system and a set of computer system for image reconstruction. It is shown in Fig. 1.

Some electrodes are arranged out of the sensor’s wall to produce the capacitance excitation among its couples. The number of produced capacitance’s measurement dimension relates to the number of N attached electrodes which is formulated as \( N(N-1)/2 \) [1]. To make sure the capacitance and electrical field measurement can be processed on the computerized imaging system, an electronics data acquisition system is set up. The Algorithm to reconstruct the ECVT signal which utilize the capacitance measurement is set up the control reconstruction computer system. The corresponding reconstructed image describes the prediction of permittivity distribution (dielectric value) inside the sensing domain.

B. The Mathematical Model

Mathematically, ECVT can be divide into two main working processes: forward problem and inverse problem.

Forward Problem

The forward problem covers the mathematical model to produce the capacitance as the input on the imaging process. The capacitance measurement process follows the Poisson Equation that can be represented on 3 dimensional space as expressed in Eq. (1) [1].

\[
\nabla \varepsilon(x,y,z) \nabla^2 \phi(x,y,z) = - \rho(x,y,z)
\]

where \( \nabla \varepsilon(x,y,z) \) represents the permittivity distribution, \( \nabla^2 \phi(x,y,z) \) represents the potential distribution of the electrical field and \( \rho(x,y,z) \) represents the charge density.

By assuming no charge inside the sensor, the Eq.1 can be represented as Eq. (2) [1].

\[
\nabla \varepsilon(x,y,z) \nabla^2 \phi(x,y,z) = 0
\]

In the Eq. (2), the potential value can be calculated using FEM (Finite Element Method). If the potential can be calculated, then the capacitance can be calculated using volume integral expressed in Eq. (3) [1].

\[
C_i = -\frac{1}{\Delta V_i} \int_A \epsilon(x,y,z) \nabla \phi(x,y,z) dA
\]

where \( \Delta V_i \) is the voltage difference between the electrodes pair and \( A_i \) is the surface area enclosing the detector’s electrode.

Equation (2) relates the perturbation inside the sensing domain which is represented by dielectric constant (permittivity) \( \epsilon(x,y,z) \), into the measured capacitance \( C_i \).

To solve Eq. (3) analytically or numerically may give more precise solution. However it will be much more complicated and computationally cost due to its complexity. Thus, linearization method, called as sensitivity model, is used to approach the solution. It is taken due to its simplicity. The linearized form of Eq. (3) can be mathematically formulated as Eq. (4) [1].

\[
C = Sg
\]

where \( C \) is \( M \)-dimension of measured capacitance vector, \( g \) is \( N \)-dimension of permittivity distribution vector and \( S \) is \( M \times N \) dimension of sensitivity matrix. The sensitivity matrix will be used as the projection matrix as expressed in Eq. (5) [1].

\[
S_{ij} \equiv V_{ij} \frac{E_{si}(x,y,z).E_{di}(x,y,z)}{V_{si}V_{di}}
\]

where \( E_{si} (= -\nabla \phi) \) is the electrical field distribution vector when the source electrode in the \( i \)th pair is activated with voltage \( V_{si} \) while the rest of the electrodes are grounded. \( E_{di} \) is the electrical field distribution vector when the detector electrode in the \( i \)th pair is activated with voltage \( V_{di} \) while the rest of the electrodes are grounded. \( V_{oj} \) is the volume of the \( j \)th voxel.

Inverse Problem

The inverse problem is the ECVT imaging that utilizes the capacitance measurement to predict the perturbation which is represented by permittivity property. As the linearization on Eq.(4) is defined, the inverse problem simply defines the prediction of \( g \) by utilizing \( C \) as expressed in Eq. (6) [1].

\[
g = S^{-1}C
\]
linear system. The underdetermined linear system becomes main problem on the imaging research area especially on the electrical tomography.

III. ECVT IMAGING BASED ON COMPRESSIVE SENSING FRAMEWORK

Compressive sensing is a framework that enable signal recovery with less data sampling compared to Shannon-Nyquist Theorem. Mathematically, it is promising for the ECVT imaging system since the dimension of measurement data is much smaller compared to the dimension of the projected data. This section discusses our proposed algorithm for ECVT imaging based on compressive sensing framework.

A. Compressive Sensing Framework

CS framework is said to be enable to reconstruct a certain naturally sparse or transformed sparse signal by utilizing less number of sampling data compared to the Shannon-Nyquist theorem [5, 8, 9]. It means that the dimension of the measurement matrix is lower or practically much lower compared to the dimension of the projected data. It will lead into underdetermined linear system which is complicated to solve [7]. CS is an approach to find the solution with some properties that should be satisfied. The properties are sparsity and incoherence between measurement matrix and the dictionary.

The Mathematical Model

Given a discrete-time signal \( \alpha \in \mathbb{R}^N \) and consider a measurement system that acquires \( M \)-dimension of measurement value, then the linear measurement can be represented as Eq. (7) [9].

\[
y = \Phi \alpha
\]

where \( \Phi \in \mathbb{R}^{M \times N} \) and \( y \in \mathbb{R}^M \). \( \Phi \) denotes the sensing or measurement matrix. \( M \) is typically is much smaller compared to \( N \). \( \alpha \in \mathbb{R}^N \) is a coefficient vector which normally has only \( K \ll N \) non-zero coefficient [9]. \( \alpha \) is needed to be reformulated to ensure that it will adapt with the CS framework [7]. The original signal \( \alpha \) is often reformulated as a linear combination of a small number of signals taken from a “resource database” determined as dictionary \( \psi \in \mathbb{R}^{N \times \psi} \) and has the formulation as expressed in Eq. (8).

\[
\alpha = \psi s
\]

On this state, \( \alpha \) is considered as sparse signal in base \( \psi \) with \( K \)-degree of sparse. Hence, the Eq. (6) can be represented as:

\[
y = \Phi \psi s
\]

The main idea of CS system is projection of \( \alpha \) to a low dimensional measurement vector \( y \) by measurement matrix \( \Phi \) which is completely has no relation to the sparse base \( \psi \) [5, 8, 9]. To perform a good recovery, the RIP (Restricted Isotropy Property) should be satisfied [5, 9]. This criterion restricts the projection matrix \( \Phi \), hence the convergence of the reconstruction algorithm can be ensured [10]. Mathematically, for any \( K \)-sparse signal \( s \) and any constant \( \varepsilon \in (0, 1) \), the RIP criterion restricts the \( \Phi, \psi \) into criteria as expressed in Eq. (10) [4, 11].

\[
1 - \varepsilon \leq \frac{||\Phi \psi s||_2}{||s||_2} \leq 1 + \varepsilon
\]

To physically proof the RIP is difficult. Thus, other parameters have to be formulated to represent the RIP criterion. These parameters measure the incoherence between the projection matrix \( \Phi \) and the dictionary \( \psi \). Theoretically, the more incoherence, the more precise the recovery is. One way to see the incoherence is Gram Matrix. For a given matrix \( B = \Phi \psi \), the Gram matrix is defined in Eq. (11) [4].

\[
G = B^T B
\]

the \((i,j)^{th}\) element of Gram matrix of \( A \) is formulated in Eq. (12).

\[
G_{ij} = B_i^T B_j
\]

The Gram matrix is normalized such that \( G_{ij} = 1, f or \forall i = j \). The mutual coherence of \( B \) is determined by the maximum value of the off-diagonal element of \( G \).

The value of the mutual coherence is bounded in the interval of \( \mu \leq \mu(B) \leq 1 \) that lower boundary \( \mu \) is defined in Eq. (13) [4, 9].

\[
\mu \equiv \frac{L - M}{\sqrt{M(L - 1)}}
\]

The performance measurement of the mutual coherence between the sensing matrix and the dictionary can be represented by the average of the mutual coherence which is formulated in Eq.(14) [4],[9].

\[
\bar{\mu}(B) = \frac{\sum_{(i,j),with \ (i \neq j)} G_{ij}}{N_i}
\]

where \( N_i \) is the number of off-diagonal elements. The distribution of the off diagonal element of Gram matrix is also can be used to analyze the incoherence between the dictionary and the sensing matrix which is also presented in this paper.

B. Proposed Algorithm for Static ECVT Imaging

We propose an algorithm for ECVT Imaging that is developed based on Compressive Sensing (CS) principle. The characteristic of ECVT imaging which is mostly sparse or at least sparse in certain transformed domain, makes CS possible to be adopted in ECVT imaging.
Projection matrix and dictionary determination is the early factors that should be concerned. In our simulation, due to the physical process of ECVT, we used the Sensitivity Matrix as the projection matrix and fixed Discrete Cosine Transform (DCT) as the dictionary. Another factor that should be concerned is the sparse recovery technique. We used $\ell_1$ optimization to reconstruct the sparse representation signal that simultaneously reconstruct the original signal.

Given the linearization of ECVT forward problem as stated in Eq. (4), the permittivity is reformulated as Eq. (15) to guarantee the adaptable into the CS framework. The permittivity $g$ is represented into another sparse signal representation using DCT dictionary as expressed in Eq. (15).

$$g = \psi \ast \theta$$  \hspace{1cm} (15)

where $g$ is $N \times 1$ array of permittivity value, $\psi$ is $N \times k$ matrix of dictionary that fulfill the DCT requirement expressed in Eq. (16).

$$\psi(p, q) = \sqrt{\frac{2}{N}} c(p) \cos \left(\frac{(2q - 1)(q - 1)}{2N}\right)$$  \hspace{1cm} (16)

where

$$p, q = 1, 2, \ldots, N, c(p) = \begin{cases} \frac{1}{\sqrt{2}} & p = 1 \\ 1, p = 2, 3, \ldots, N \end{cases}$$

and $\alpha$ is $N \times 1$ array of the sparse representation signal.

Using Eq. (14), the ECVT imaging forward problem stated in Eq. (4) can be reformulated into Eq. (17).

$$C = S \psi \theta$$  \hspace{1cm} (17)

The inverse problem stated in Eq. (6) becomes Eq. (18).

$$\theta = (S \psi)^{-1} C$$  \hspace{1cm} (18)

The proposed algorithm is comprehensively described in the following pseudo code.

**Algorithm 1. CS with $\ell_1$ sparse recovery**

**Purpose:** ECVT Image Reconstruction  
**Input:** Capacitance, Sensitivity Matrix, Dictionary  
**Output:** Permittivity Distribution

**Module 1**  
Data Acquisition (Capacitance Measurement)

**Module 2**  
Sensitivity Matrix (Projection Matrix) construction  
Projection matrix on ECVT is constructed based on Eq. (5)

**Module 3**  
Sparse Signal Representation recovery  
**Initialization**  
- Initial Projection matrix $S$  
- Sparse base (dictionary) $\psi$  
- Calculate the equivalent dictionary $D = S \ast \psi$

**Iteration Procedure**  
Sparse Signal ($\theta$) recovery by $\ell_1$ optimization

**Module 4**  
Original Signal Recovery  
Once the sparse signal representation succeeds to recover, the original signal (permittivity value) can be recovered simultaneously based on Eq. (15)

**C. Simulation Set Up**

A simulation is set up to evaluate the performance of the proposed ECVT imaging algorithm based on Compressive Sensing framework with $\ell_1$ sparse recovery. The measurement of ECVT is performed using cylindrical geometry sensors composed by 8 electrodes as illustrated in Fig. 2. An object that has significant dielectric contrast to the environment is located in the center of the sensor. The imaging algorithm detects the existence of the object.

![Cylindrical Sensor with 8 electrodes, d=10 cm, h=20 cm](Source [6] CTech Lab)

**Table 1. Properties of Simulation**

<table>
<thead>
<tr>
<th>Simulation’s Properties</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Geometry</td>
<td>Cylindrical Sensor with 8 electrodes</td>
</tr>
<tr>
<td>Detected object</td>
<td>Ball with r=4cm</td>
</tr>
<tr>
<td>Contrast dielectric</td>
<td>1:3; 1:6; 1:80</td>
</tr>
<tr>
<td>Detected object position</td>
<td>Center of the sensor</td>
</tr>
<tr>
<td>Number of object detected</td>
<td>Single object</td>
</tr>
<tr>
<td>Performance measurement parameter</td>
<td>Qualitative measurement, NMSE, R</td>
</tr>
</tbody>
</table>

Figure 3 describes the simulation flow to evaluate the performance of the proposed ECVT imaging algorithm based on CS framework.
IV. RESULT AND DISCUSSION

To evaluate the performance of CS with $\ell_1$ sparse recovery into ECVT imaging, a set of simulation had been performed. A simulated ECVT sensor is designed to imitate the sensing environment. The simulated sensor in cylindrical geometry is arranged using 8 electrodes in 4 rows of sensors. Thus, there are 28 possible capacitance measurement values (nC2). The capacitance measurement is used to project the perturbation inside the sensing domain which is represented by permittivity distribution. The permittivity distribution is projected into 20 × 20 × 20 voxels.

A sphere formed object is used as the testing object inside the cylindrical sensor. We evaluated the proposed algorithm to detect the object. The accuracy level is measured by the coefficient of correlation (R) and Normalized Mean Square Error (NMSE). We applied the qualitative measurement as additional assessment. As benchmarks, ILBP is used as the reference of the CS-$\ell_1$ performance.

The results of the performance of ILBP and CS-$\ell_1$ correspond to qualitative measurement for various dielectric contrast (1:3, 1:6, 1:80) are shown in Fig. 5-10, respectively. Based on visual analysis, it can be seen that CS-$\ell_1$ excel the ILBP performance in all dielectric contrast since it eliminates the elongation error, especially for high dielectric contrast (as shown in Fig. 9 and Fig. 10). On each figure, the imageries shown in (b) showed the 20 imaging slices from top to bottom of the column profiles of the reconstruction results. In the original image, the sphere object is detected from slice 7 to slice 14. However, the ECVT imaging reconstruction using ILBP, the object is detected since the first slice to the last one (1-20). The proposed algorithm succeeds to reduce the elongation error up to 83% without presenting artefact in the edge of the sensor as presented on CS-OMP [7].

To analyze the accuracy level of the measurements, the coefficient correlation R and NMSE are presented graphically in Fig. 11. It showed that the CS-$\ell_1$ performs better compared to ILBP significantly. In the low dielectric contrast, the correlation of the predicted imaging to the original object is improved up to 0.22 point.
The proposed algorithm CS-$\ell_1$ succeeds to perform better on ECVT imaging compared to ILBP based on the visual and quantitative measurement. However, based on the visual observation, the reconstructed imaging using our algorithm is still limited to obtain the real object’s shape. It should produce perfectly sphere rather than ellipsoid.

Analysis of Gram Matrix

High incoherency between the measurement matrix and the sparse base representation (dictionary) lead to better reconstruction or recovery. It represents the Restricted Isotropy Property (RIP) requirement. We used the component off-diagonal distribution of the Gram matrix to measure the RIP condition of the CS design on the ECVT imaging. The less or the absence of value distribution on the right hand side of the figure (closed to 1) the more precise the reconstruction is. Gram matrix $A$ is defined as follow:

$$ A = (S\psi)^T (S\psi) $$  \hspace{1cm} (19)

Figure 12 presents the distribution of the off-diagonal component on Gram matrix $A$.

The calculation showed that most of the off-diagonal values are concentrated to the left hand side on the Fig.12. However, it still contains quite significant distribution on the right hand side of the figure which is closed to 1. It indicates that the performance of CS design for ECVT imaging could be improved. In the future, we will apply the optimization method, either in the measurement matrix or in the dictionary design, that theoretically will lead to better reconstruction, which still could not be achieved using the proposed algorithm.

![Figure 12. Component off-diagonal distribution (Gram Matrix)](image)

V. CONCLUSION

Compressive Sensing with $\ell_1$ sparse recovery has been proposed for ECVT imaging. A set of simulation has been performed to evaluate the performance of the proposed algorithm. Cylindrical sensors with 8 electrodes are used as sensor geometry and a sphere object is located in the center of the sensor. The proposed algorithm was able to detect the form of the object and it is evaluated based on its comparison with the existing ILBP imaging algorithm.

The simulation results showed impressive performance on this early CS frame work design for ECVT imaging. CS with $\ell_1$ sparse recovery succeeds to eliminate the elongation error that is normally occurred on ECVT imaging using ILBP. The accuracy based on coefficient of correlation (R) and Normalized Mean Square Error (NMSE) are also analyzed. However, the resulted imaging still produces elliptical shape instead of perfect sphere as the original object’s shape.

The advantage of the proposed algorithm is the possibility to reduce the number of electrodes. CS framework enables to recover signal with less number of sampling compared to Shannon-Nyquist theorem. Thus, it will be very potential to have higher accuracy level in ECVT imaging, by analyzing only a few data on accessible capacitance measurement. This will also imply into efficiency of the hardware design.

The proposed algorithm is very potential to be developed further for higher ECVT imaging accuracy. However, it needs to be tested in more complicated study cases and also needs more experimental data.

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