Digital Communication
Transfer function, interleaving, concatenated codes
Lecture - 10

Ir. Muhamad Asvial, MSc., PhD

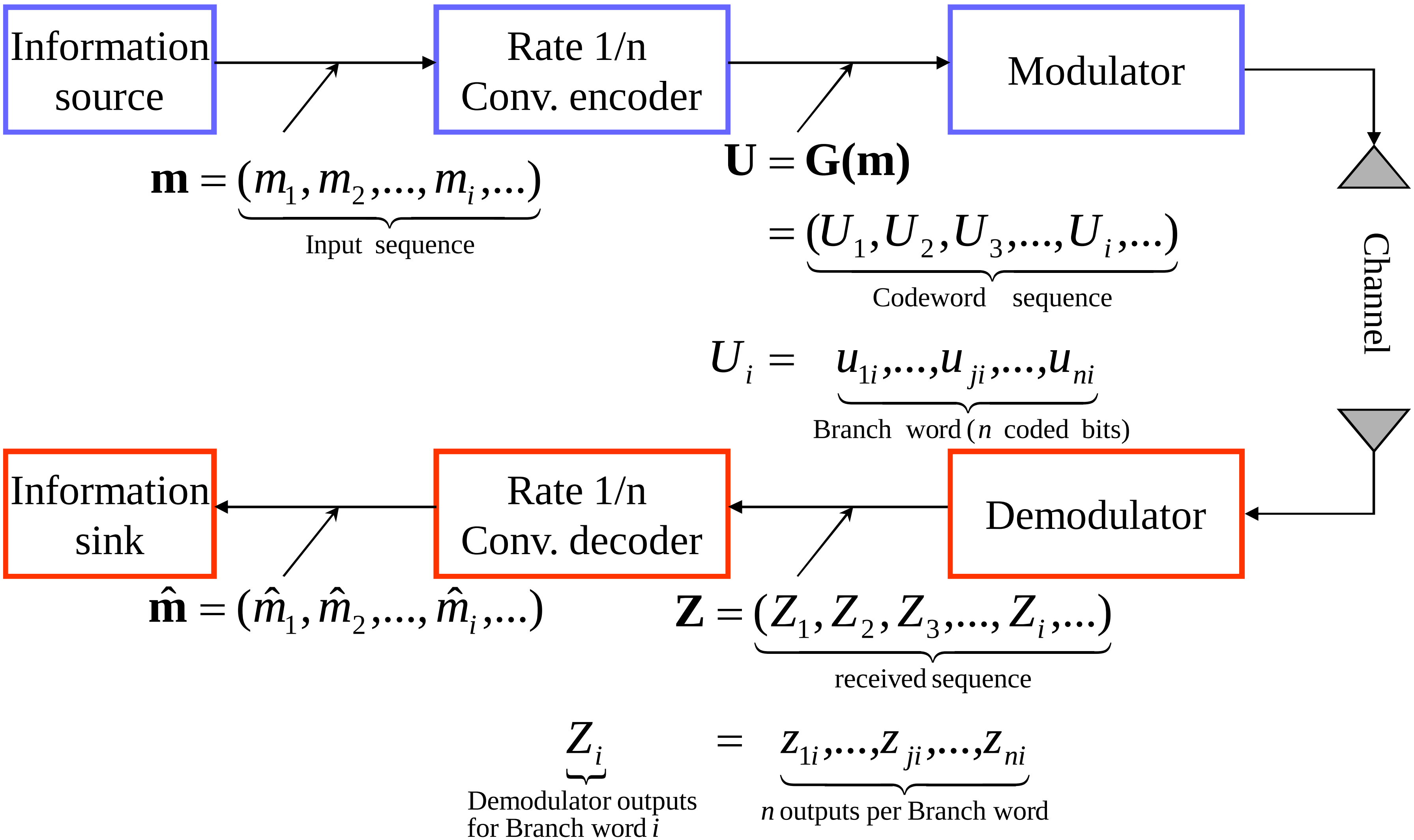
Center for Information and Communication Engineering Research (CICER)

Electrical Engineering Department - University of Indonesia

E-mail: asvial@ee.ui.ac.id

http://www.ee.ui.ac.id/cicer

Block diagram of the DCS



State diagram

- A finite-state machine only encounters a finite number of states.
- State of a machine: the smallest amount of information that, together with a current input to the machine, can predict the output of the machine.
- In a Convolutional encoder, the state is represented by the content of the memory.
- Hence, there are 2^{K-1} states.

State diagram – cont'd

- A state diagram is a way to represent the encoder.
- A state diagram contains all the states and all possible transitions between them.
- Only two transitions initiating from a state
- Only two transitions ending up in a state

Soft and hard decisions

- In hard decision:
  - The demodulator makes a firm or hard decision whether one or zero is transmitted and provides no other information for the decoder such that how reliable the decision is.
  - Hence, its output is only zero or one (the output is quantized only to two level) which are called “hard-bits”.
- Decoding based on hard-bits is called the “hard-decision decoding”.

Soft and hard decision decoding

- In hard decision:
  - The demodulator makes a firm or hard decision whether one or zero is transmitted and provides no other information for the decoder such that how reliable the decision is.

- In Soft decision:
  - The demodulator provides the decoder with some side information together with the decision. The side information provides the decoder with a measure of confidence for the decision.

Soft and hard decision decoding …

- ML soft-decisions decoding rule:
  - Choose the path in the trellis with minimum Euclidean distance from the received sequence

- ML hard-decisions decoding rule:
  - Choose the path in the trellis with minimum Hamming distance from the received sequence

The Viterbi algorithm

- The Viterbi algorithm performs Maximum likelihood decoding.

- It finds a path through trellis with the largest metric (maximum correlation or minimum distance).
  - At each step in the trellis, it compares the partial metric of all paths entering each state, and keeps only the path with the largest metric, called the survivor, together with its metric.

Free distance of Convolutional codes

- Distance properties:
  - Since a Convolutional encoder generates codewords with various sizes (as opposite to the block codes), the following approach is used to find the minimum distance between all pairs of codewords:
    - Since the code is linear, the minimum distance of the code is the minimum distance between each of the codewords and the all-zero codeword.
    - This is the minimum distance in the set of all arbitrary long paths along the trellis that diverge and remerge to the all-zero path.
    - It is called the minimum free distance or the free distance of the code, denoted by d_{free} or d_f

Transfer function of Convolutional codes

- Transfer function:
  - Transfer function of generating function is a tool which provides information about the weight distribution of the codewords.
    - The weight distribution specifies weights of different paths in the trellis (codewords) with their corresponding lengths and amount of information.

$$T(D,L,N) = \sum_{i=d_f} \sum_{j=K} \sum_{l=1} D^i L^j N^l$$

$D, L, N:$ place holders

$i:$ distance of the path from the all-zero path

$j:$ number of branches that the path takes until it remerges to the all-zero path

$l:$ weight of the information bits corresponding to the path

Performance bounds …

- Bounds on bit error probability for memoryless channels:
  - Hard-decision decoding:

$$P_B \le \left.\frac{dT(D,L,N)}{dN}\right|_{N=1,L=1,D=2\sqrt{p(1-p)}}$$

  - Soft decision decoding on AWGN channels using BPSK

$$P_B \le Q\left(\sqrt{2d_f \frac{E_c}{N_0}}\right)\exp\left(d_f \frac{E_c}{N_0}\right)\left.\frac{dT(D,L,N)}{dN}\right|_{N=1,L=1,D=\exp(-E_c/N_0)}$$

Interleaving

- Convolutional codes are suitable for memoryless channels with random error events.

- Some errors have bursty nature:
  - Statistical dependence among successive error events (time-correlation) due to the channel memory.
    - Like errors in multipath fading channels in wireless communications, errors due to the switching noise, …

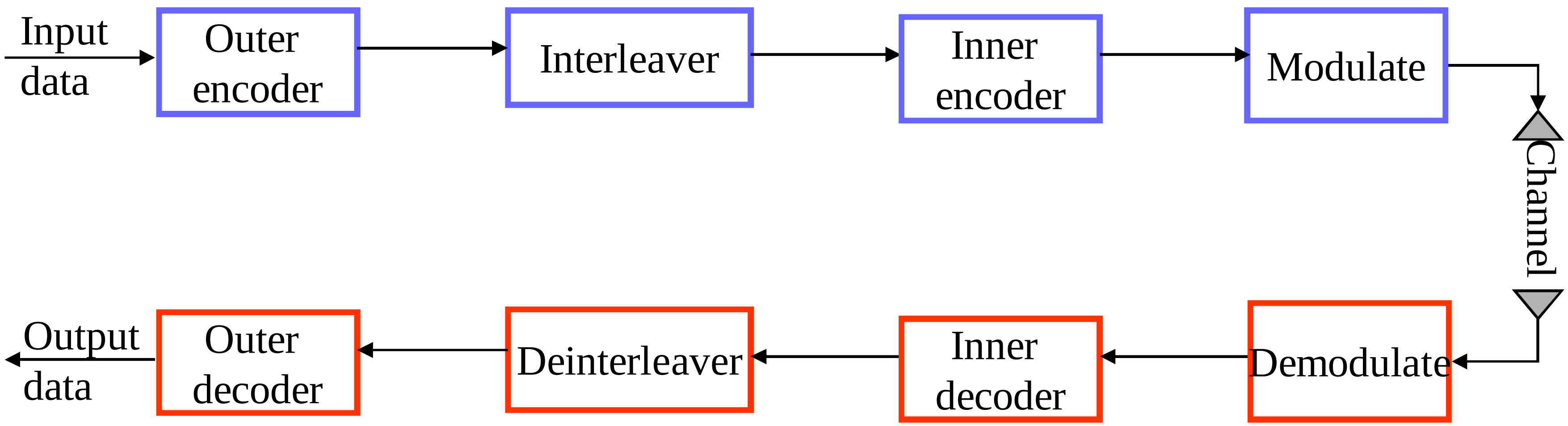
- “Interleaving” makes the channel looks like as a memoryless channel at the decoder.

Interleaving …

- Interleaving is done by spreading the coded symbols in time (interleaving) before transmission.
- The reverse in done at the receiver by deinterleaving the received sequence.
- “Interleaving” makes bursty errors look like random. Hence, Conv. codes can be used.
- Types of interleaving:
  - Block interleaving
  - Convolutional or cross interleaving

Concatenated codes

- A concatenated code uses two levels on coding, an inner code and an outer code (higher rate).
  - Popular concatenated codes: Convolutional codes with Viterbi decoding as the inner code and Reed-Solomon codes as the outer code
- The purpose is to reduce the overall complexity, yet achieving the required error performance.



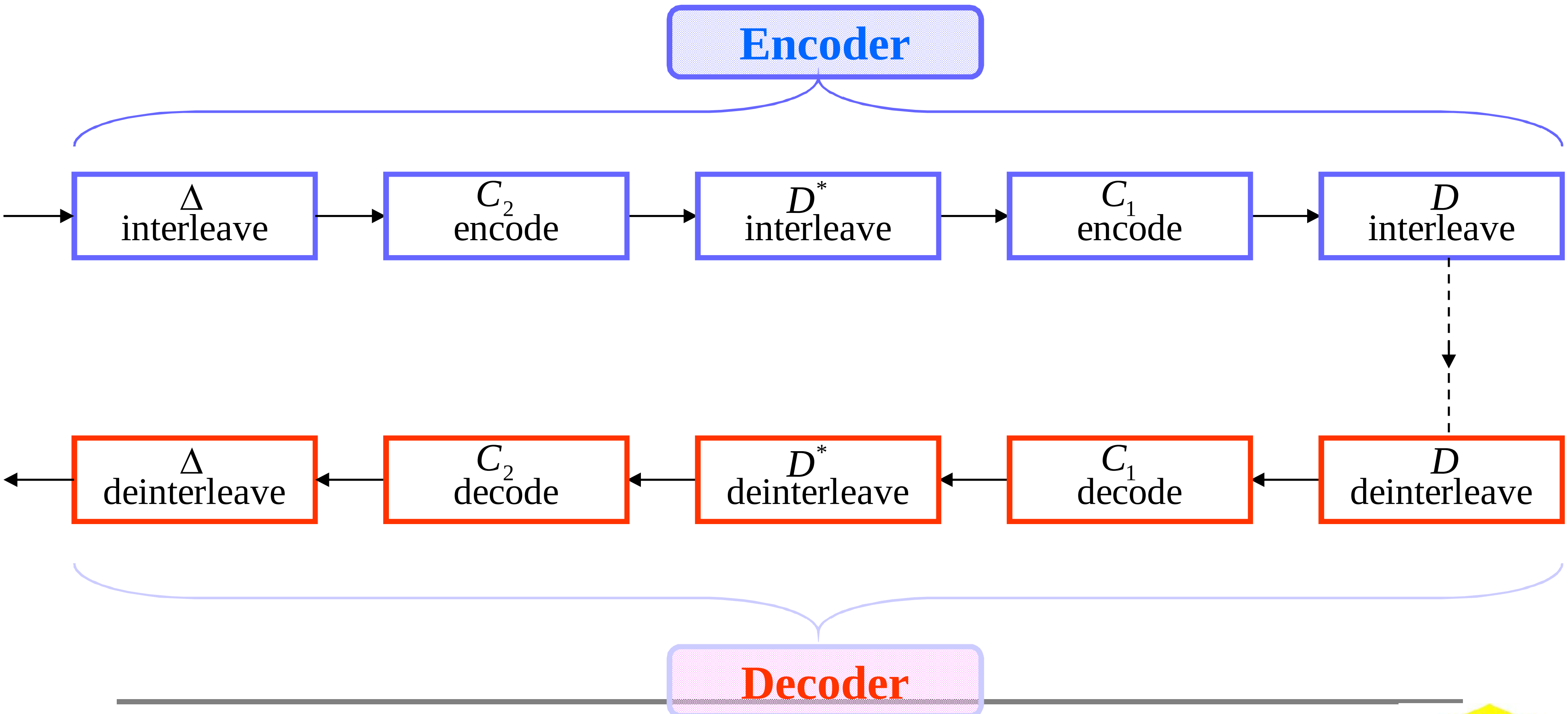
Practical example: Compact disc

"Without error correcting codes, digital audio would not be technically feasible."

- Channel in a CD playback system consists of a transmitting laser, a recorded disc and a photo-detector.
- Sources of errors are manufacturing damages, fingerprints or scratches
- Errors have bursty like nature.
- Error correction and concealment is done by using a concatenated error control scheme, called cross-interleaver Reed-Solomon code (CIRC).

Compact disc – cont'd

- CIRC encoder and decoder:



Questions?