Digital Communication
Inter symbol interference, coherent and non-coherent detection
Lecture- 6 and 7

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Bandpass modulation

- Bandpass modulation: The process of converting data signal to a sinusoidal waveform where its amplitude, phase or frequency, or a combination of them, is varied in accordance with the transmitting data.
- Bandpass signal:

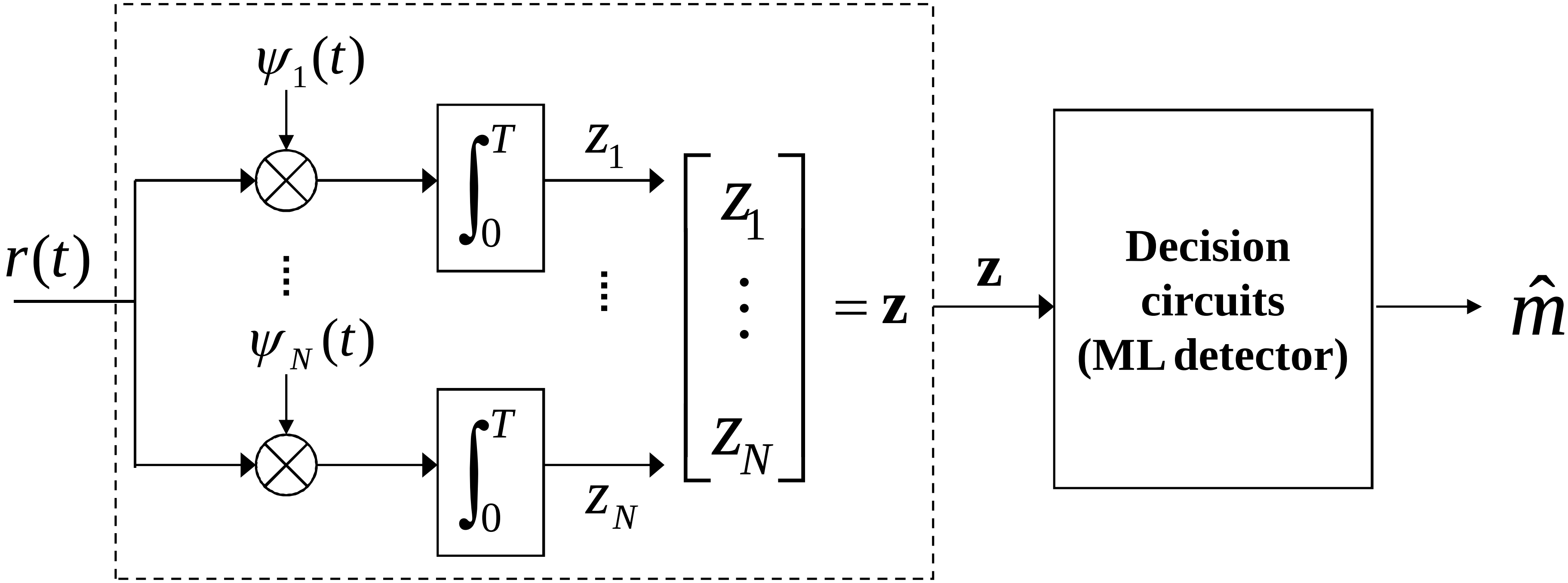
$$s_i(t) = g_T(t)\sqrt{\frac{2E_i}{T}}\cos\left(\omega_c t + (i-1)\Delta\omega t + \phi_i(t)\right) \quad 0 \le t \le T$$

where $g_T(t)$ is the baseband pulse shape with energy E_g .

- We assume here (otherwise will be stated):
  - $g_T(t)$ is a rectangular pulse shape with unit energy.
  - Gray coding is used for mapping bits to symbols.
  - E_s denotes average symbol energy given by $E_s = \frac{1}{M}\sum_{i=1}^{M} E_i$

Demodulation and detection

- **Demodulation:** The receiver signal is converted to baseband, filtered and sampled.
- **Detection:** Sampled values are used for detection using a decision rule such as ML detection rule.

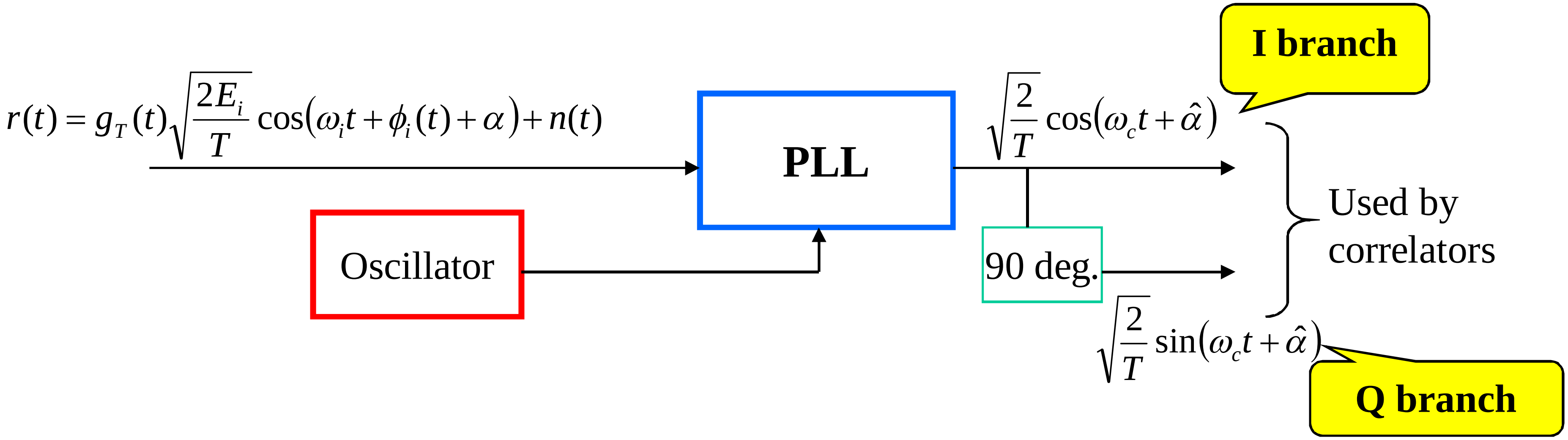


Coherent detections

- Coherent detection
  - requires carrier phase recovery at the receiver and hence, circuits to perform phase estimation.
  - Source of carrier-phase mismatch at the receiver:
    - Propagation delay causes carrier-phase offset in the received signal.
    - The oscillators at the receiver which generate the carrier signal, are not usually phased locked to the transmitted carrier.

Coherent detection ..

– Circuits such as Phase-Locked-Loop (PLL) are implemented at the receiver for carrier phase estimation ($\alpha \approx \hat{\alpha}$).



Bandpass Modulation Schemes

- One dimensional waveforms
  - Amplitude Shift Keying (ASK)
  - M-ary Pulse Amplitude Modulation (M-PAM)
- Two dimensional waveforms
  - M-ary Phase Shift Keying (M-PSK)
  - M-ary Quadrature Amplitude Modulation (M-QAM)
- Multidimensional waveforms
  - M-ary Frequency Shift Keying (M-FSK)

One dimensional modulation, demodulation and detection

- Amplitude Shift Keying (ASK) modulation:

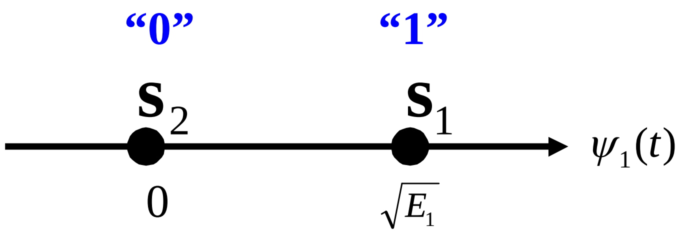
$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(\omega_c t + \phi)$$

$$s_i(t) = a_i \psi_1(t) \quad i = 1, \ldots, M$$

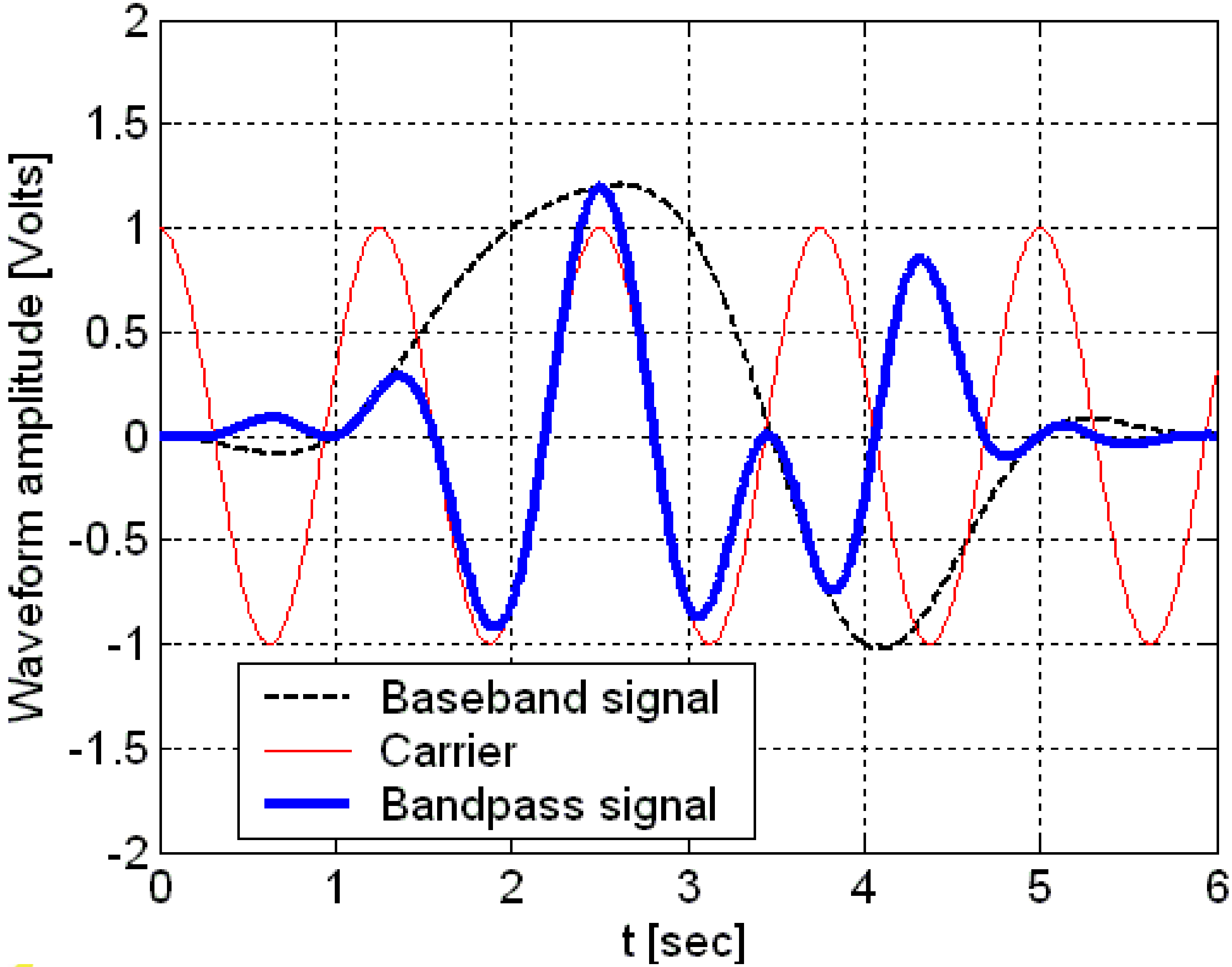
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_c t + \phi)$$

$$a_i = \sqrt{E_i}$$

On-off keying:

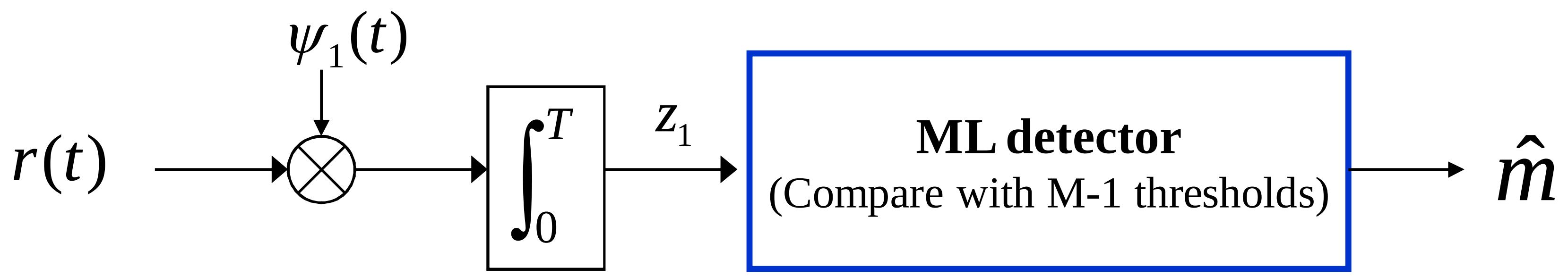


Example of bandpass modulation: Binary PAM



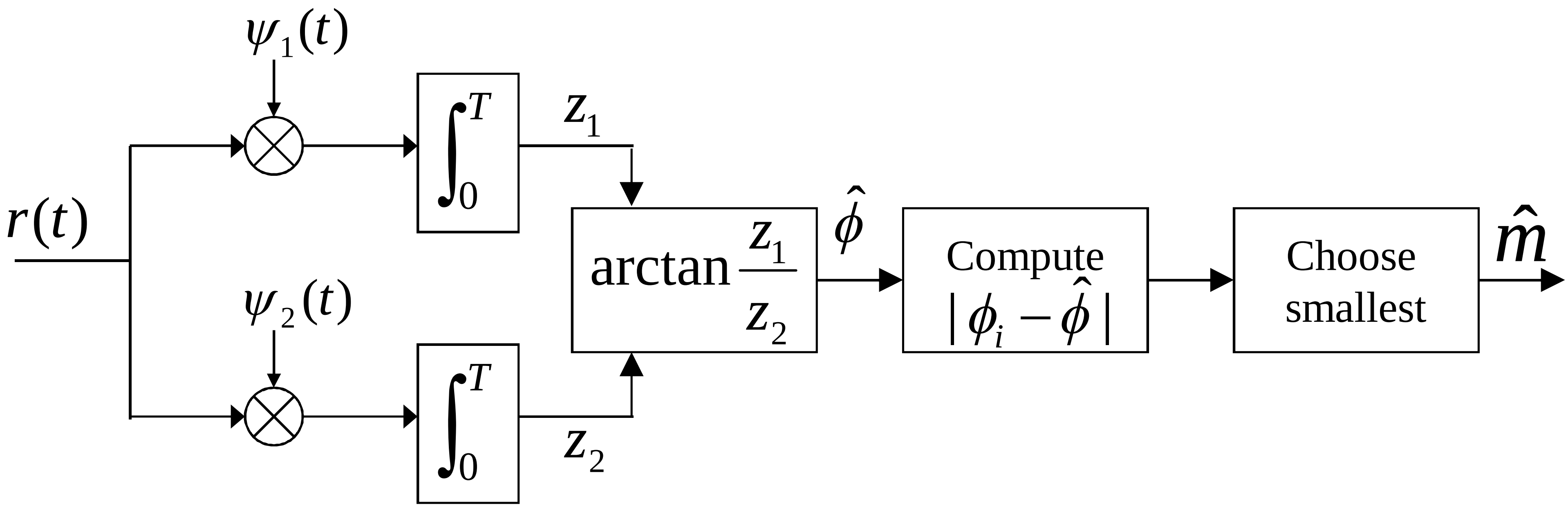
One dimensional mod.,...–cont'd

- Coherent detection of M-PAM



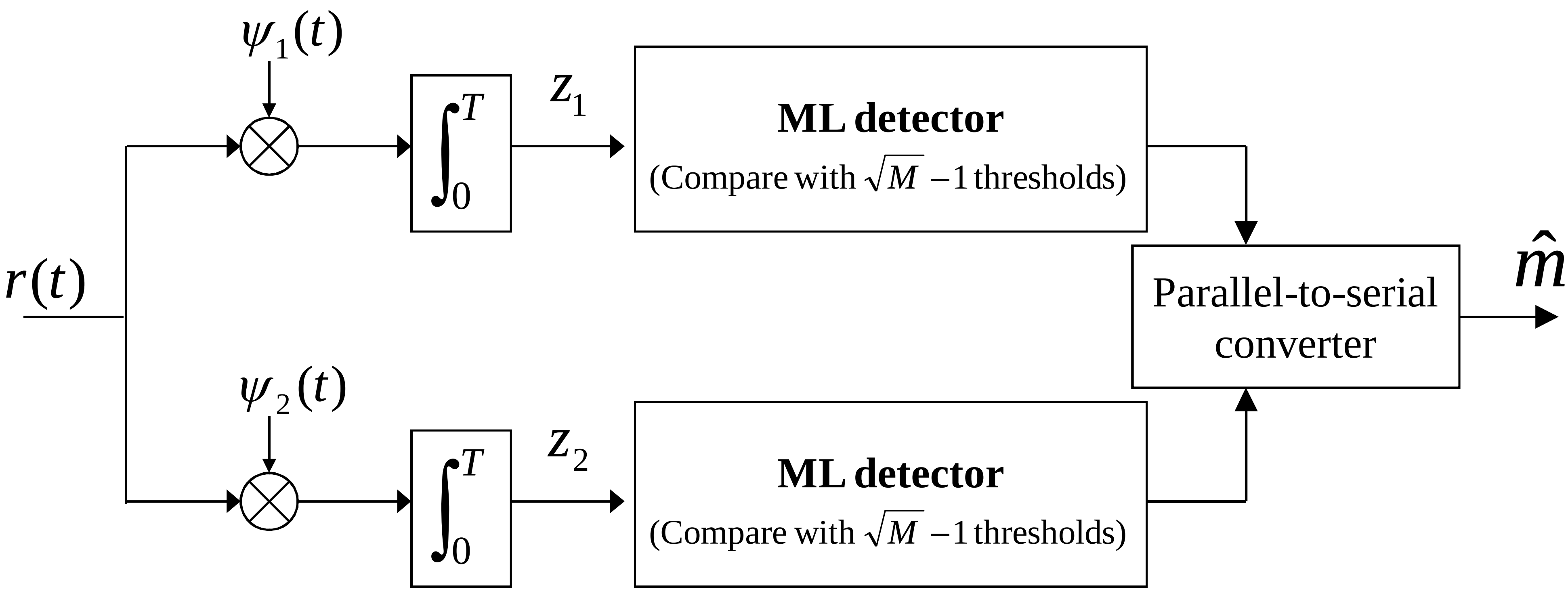
Two dimensional mod.,…(MPSK)

- Coherent detection of MPSK

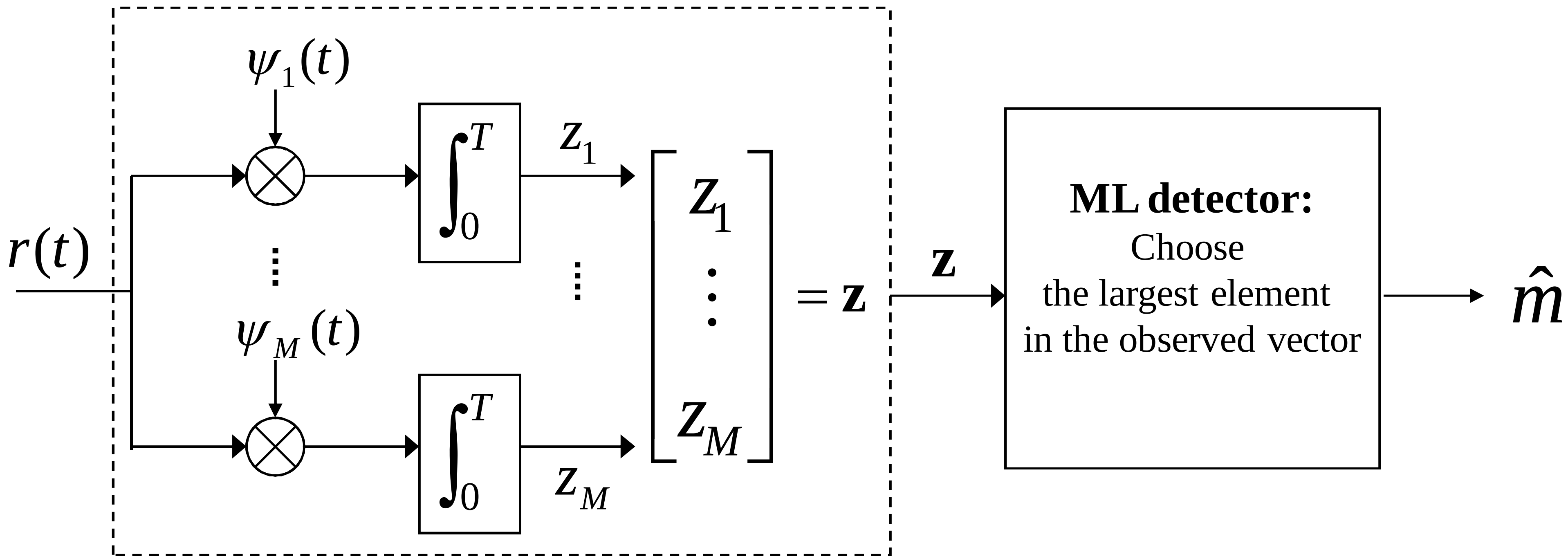


Two dimensional mod.,…. (M-QAM)

- Coherent detection of M-QAM



Multi-dimensional mod.,….(M-FSK)

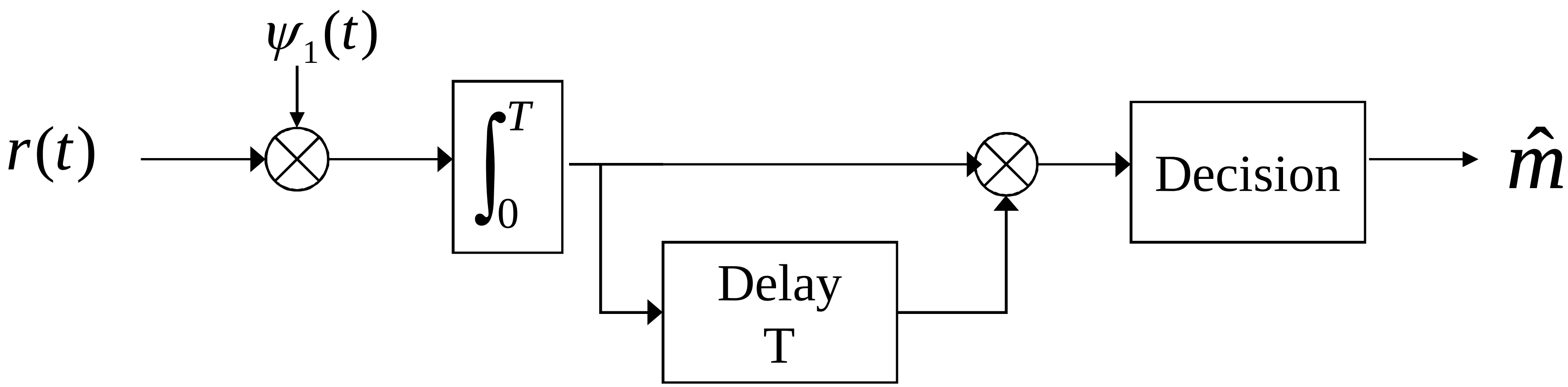


Non-coherent detection

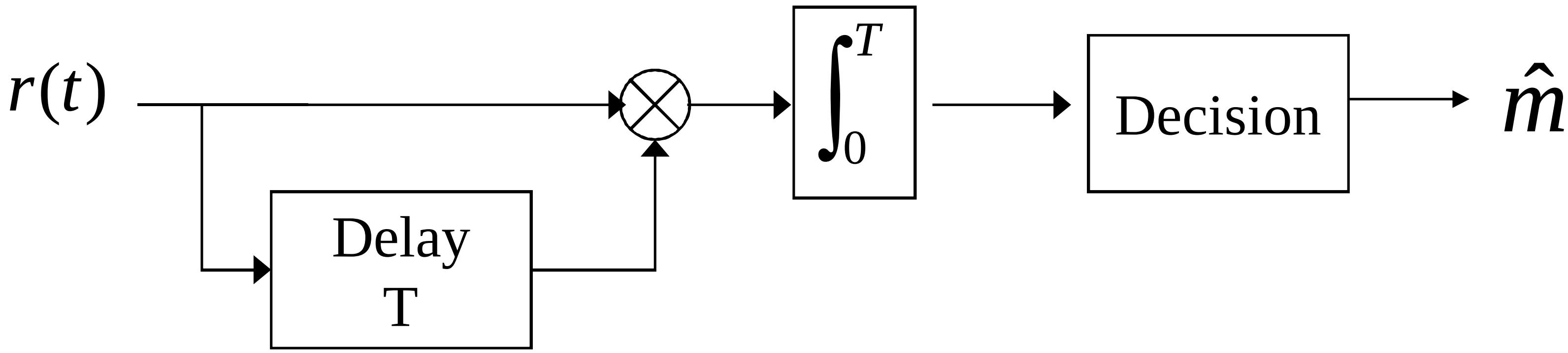
- Non-coherent detection:
  - No need in a reference in phase with the received carrier
  - Less complexity as compared to coherent detection at the price of higher error rate.

Non-coherent detection …

- Optimum differentially coherent detector



- Sub-optimum differentially coherent detector



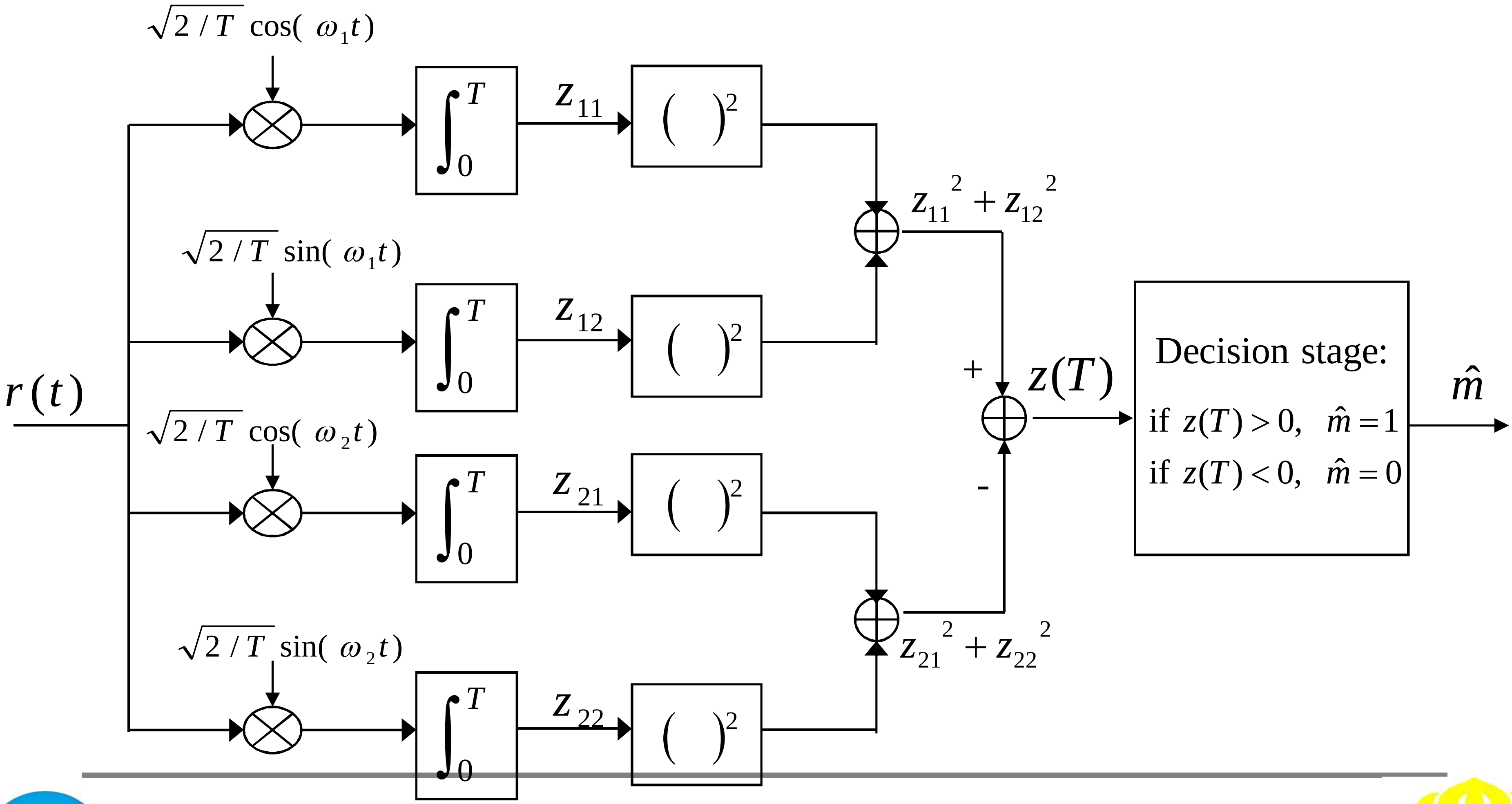
  » Performance degradation about 3 dB by using sub-optimum detector

Non-coherent detection …

- Energy detection
  - Non-coherent detection for orthogonal signals (e.g. M-FSK)
    - Carrier-phase offset causes partial correlation between I and Q braches for each candidate signal.
    - The received energy corresponding to each candidate signal is used for detection.

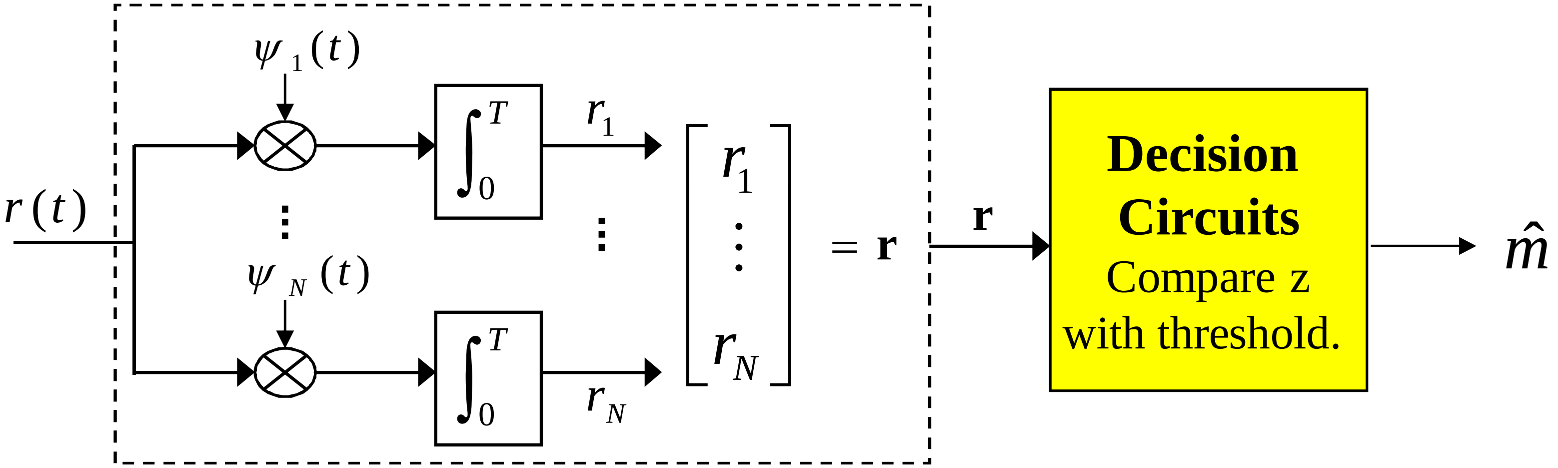
Non-coherent detection …

- Non-coherent detection of BFSK



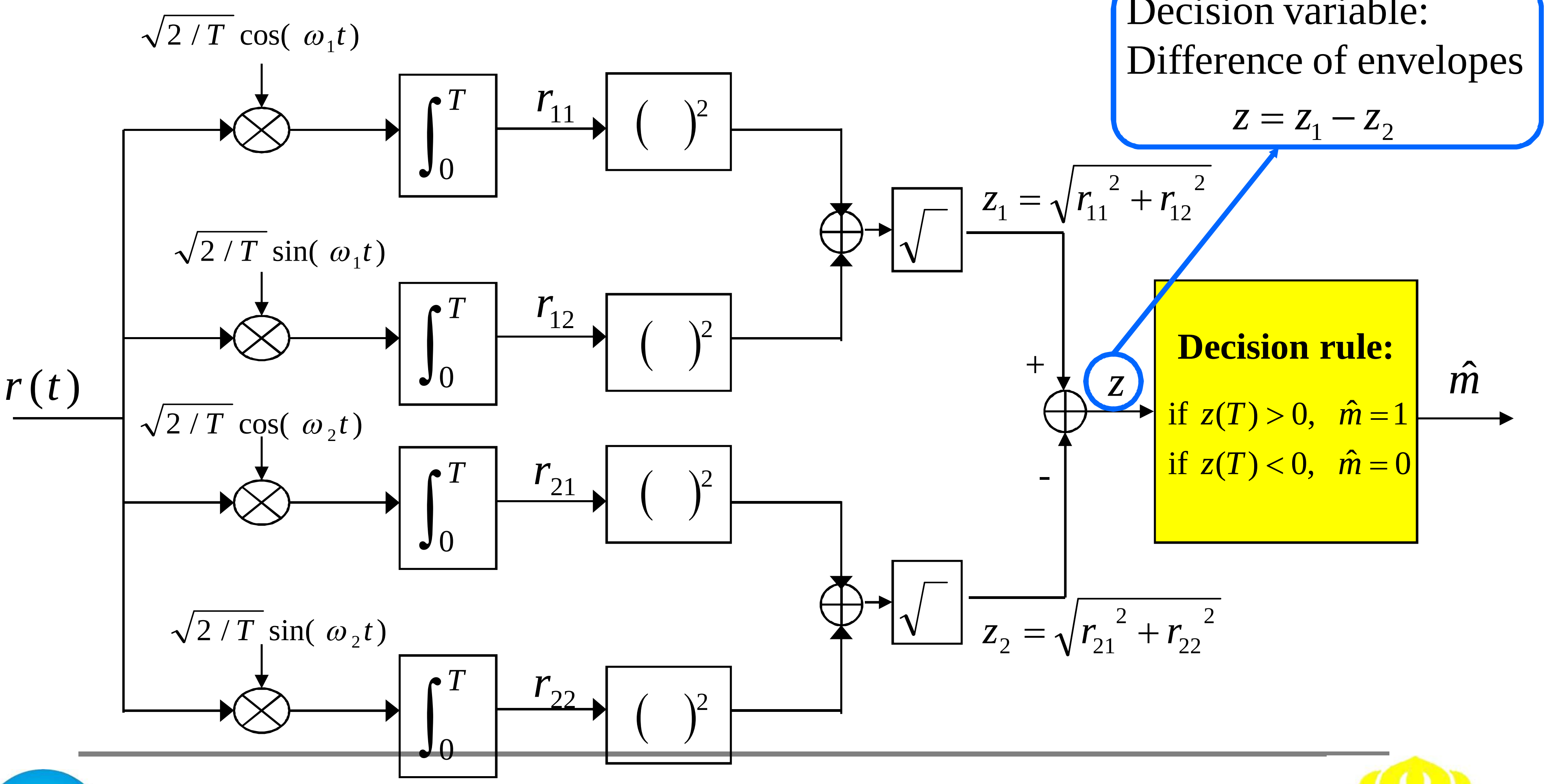
Error probability of bandpass modulation

- Before evaluating the error probability, it is important to remember that:
  - Type of modulation and detection (coherent or non-coherent), determines the structure of the decision circuits and hence the decision variable, denoted by z.
  - The decision variable, z, is compared with M-1 thresholds, corresponding to M decision regions for detection purposes.



Error probability …

- Non-coherent detection of BFSK



Error probability …

- Coherent detection of M-QAM …
- M-QAM can be viewed as the combination of two $\sqrt{M}-\mathrm{PAM}$ modulations on I and Q branches, respectively.
- No error occurs if no error is detected on either I and Q branches. Hence:
- Considering the symmetry of the signal space and orthogonality of I and Q branches:

$$P_E(M)=1-P_C(M)=1-\Pr(\text{no error detected on I and Q branches})$$

$$\Pr(\text{no error detected on I and Q branches})=\Pr(\text{no error on I})\Pr(\text{no error on Q})$$

$$=\Pr(\text{no error on I})^2=\left(1-P_E\left(\sqrt{M}\right)\right)^2$$

$$P_E(M)=4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log_2 M}{M-1}\frac{E_b}{N_0}}\right)$$

Average probability of symbol error for $\sqrt{M}-\mathrm{PAM}$

Error probability …

- Coherent detection of MPSK …
- The detector compares the phase of observation vector to M-1 thresholds.
- Due to the circular symmetry of the signal space, we have:

$$P_E(M) = 1 - P_C(M) = 1 - \frac{1}{M}\sum_{m=1}^{M} P_c(\mathbf{s}_m) = 1 - P_c(\mathbf{s}_1) = 1 - \int_{-\pi/M}^{\pi/M} p_{\hat{\phi}}(\phi)\, d\phi$$

where

$$p_{\hat{\phi}}(\phi) \approx \sqrt{\frac{2}{\pi}\frac{E_s}{N_0}}\cos(\phi)\exp\left(-\frac{E_s}{N_0}\sin^2\phi\right); \quad |\phi| \le \frac{\pi}{2}$$

- It can be shown that

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right) \quad \text{or} \quad P_E(M) \approx 2Q\left(\sqrt{\frac{2(\log_2 M)E_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

Error probability …

- Coherent detection of M-FSK …
- The dimensionality of signal space is M. An upper bound for average symbol error probability can be obtained by using union bound. Hence

$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

or, equivalently

$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{(\log_2 M)E_b}{N_0}}\right)$$

Bit error probability versus symbol error probability

- Number of bits per symbol $k = \log_2 M$
- For orthogonal M-ary signaling (M-FSK)

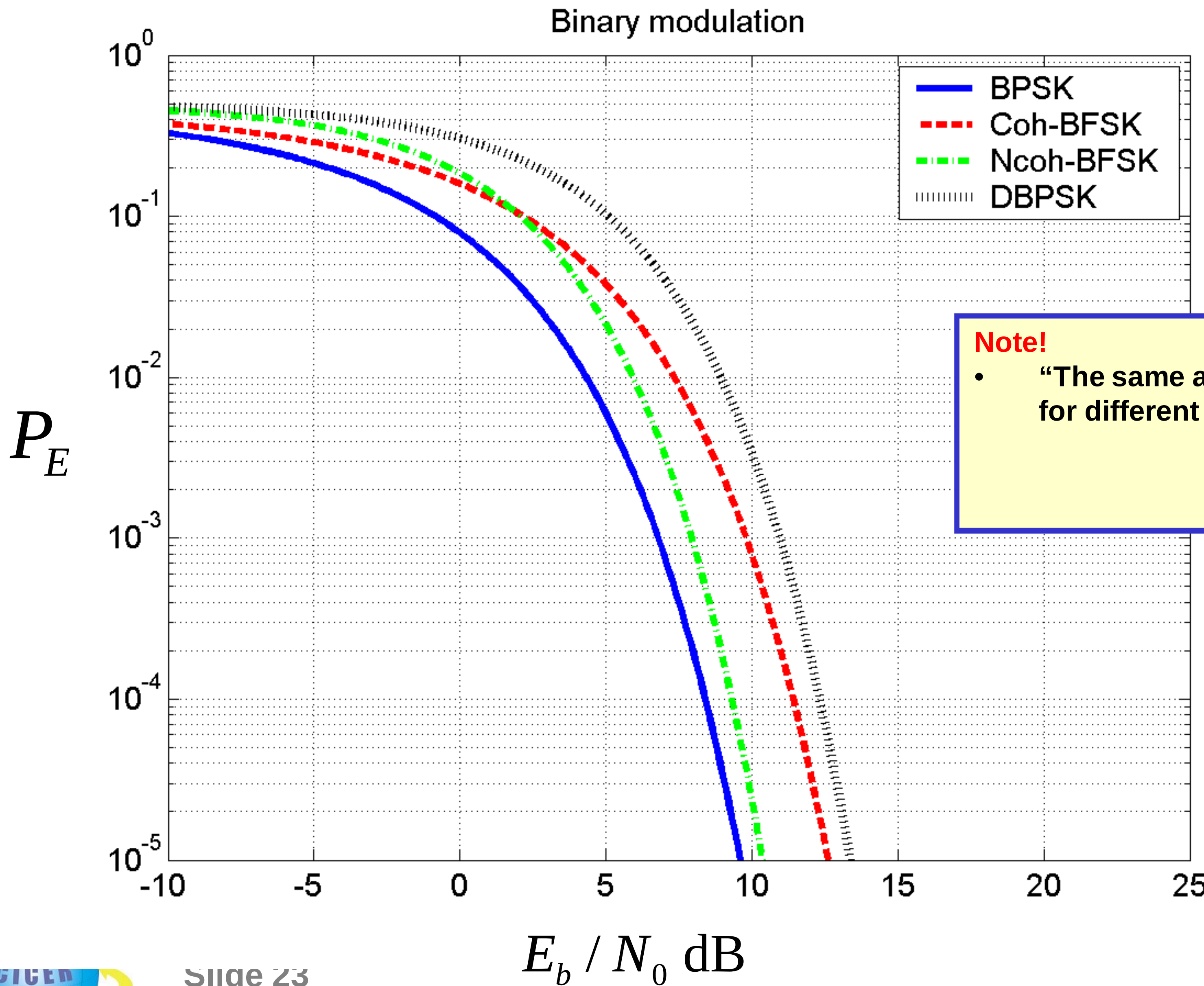
$$\frac{P_B}{P_E} = \frac{2^{k-1}}{2^k - 1} = \frac{M/2}{M-1}$$

$$\lim_{k\to\infty} \frac{P_B}{P_E} = \frac{1}{2}$$

- For M-PSK, M-PAM and M-QAM

$$P_B \approx \frac{P_E}{k} \quad \text{for } P_E << 1$$

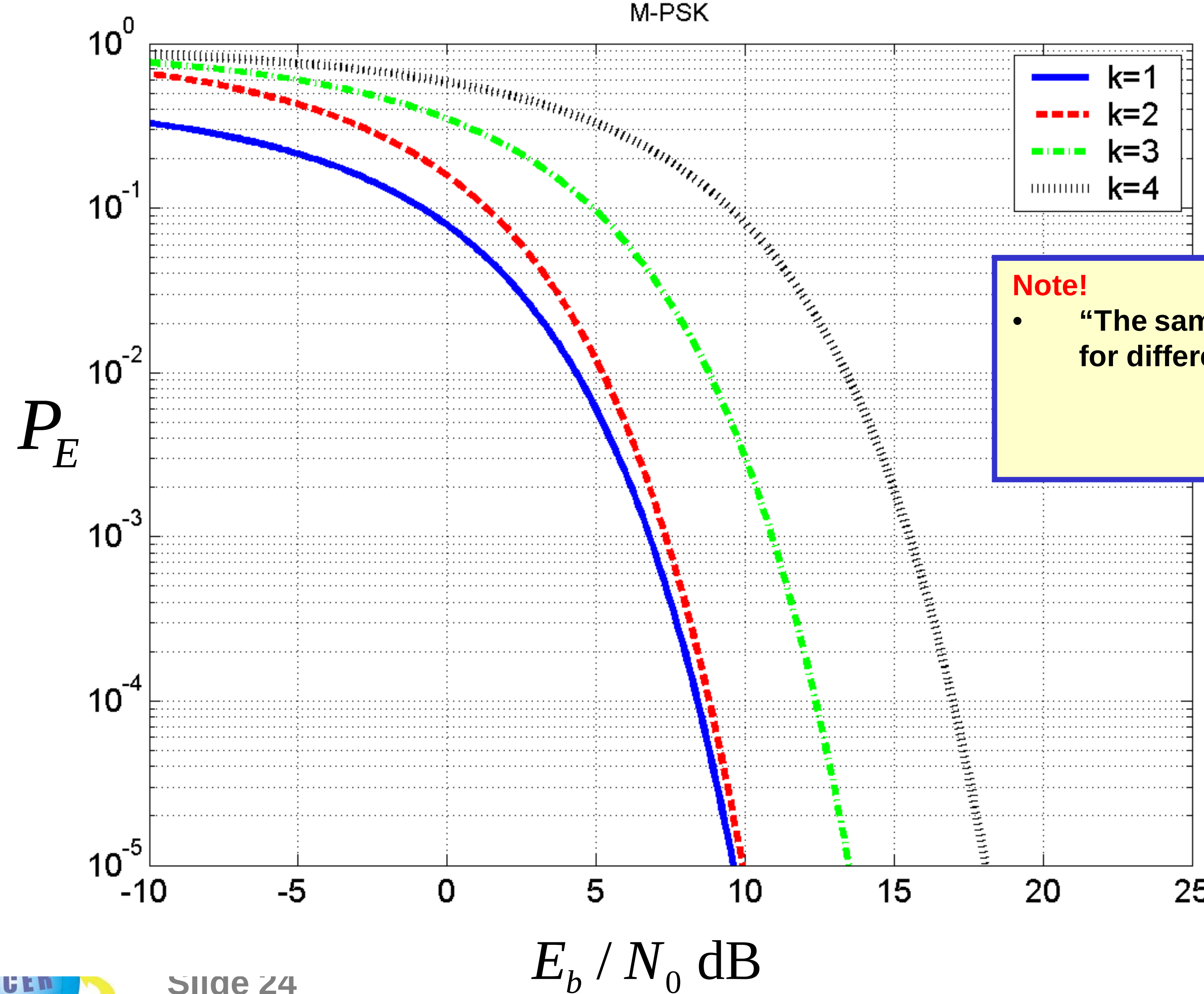
Probability of symbol error for binary modulation



Note!

- **"The same average symbol energy for different sizes of signal space"**

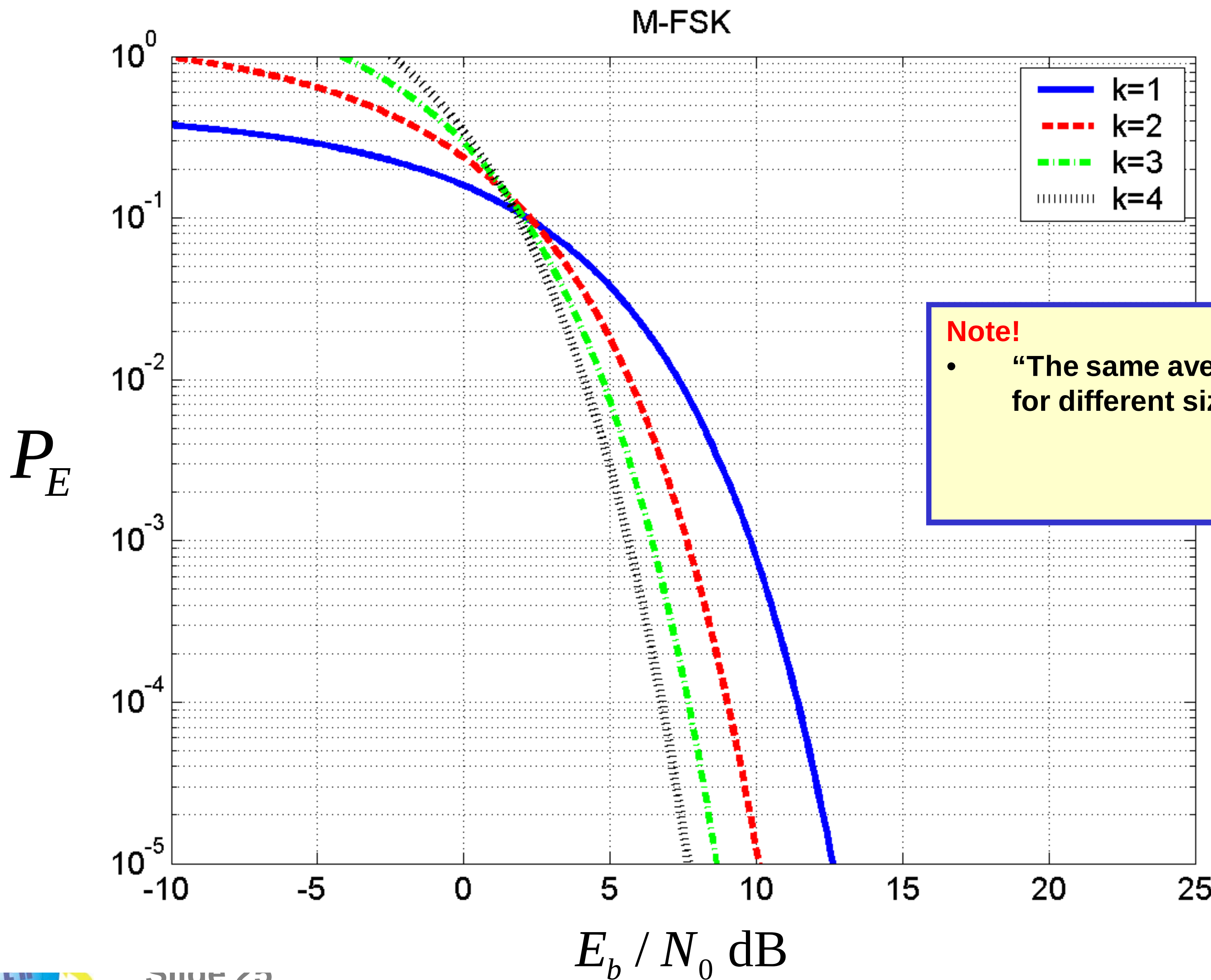
Probability of symbol error for M-PSK



Note!

- **"The same average symbol energy for different sizes of signal space"**

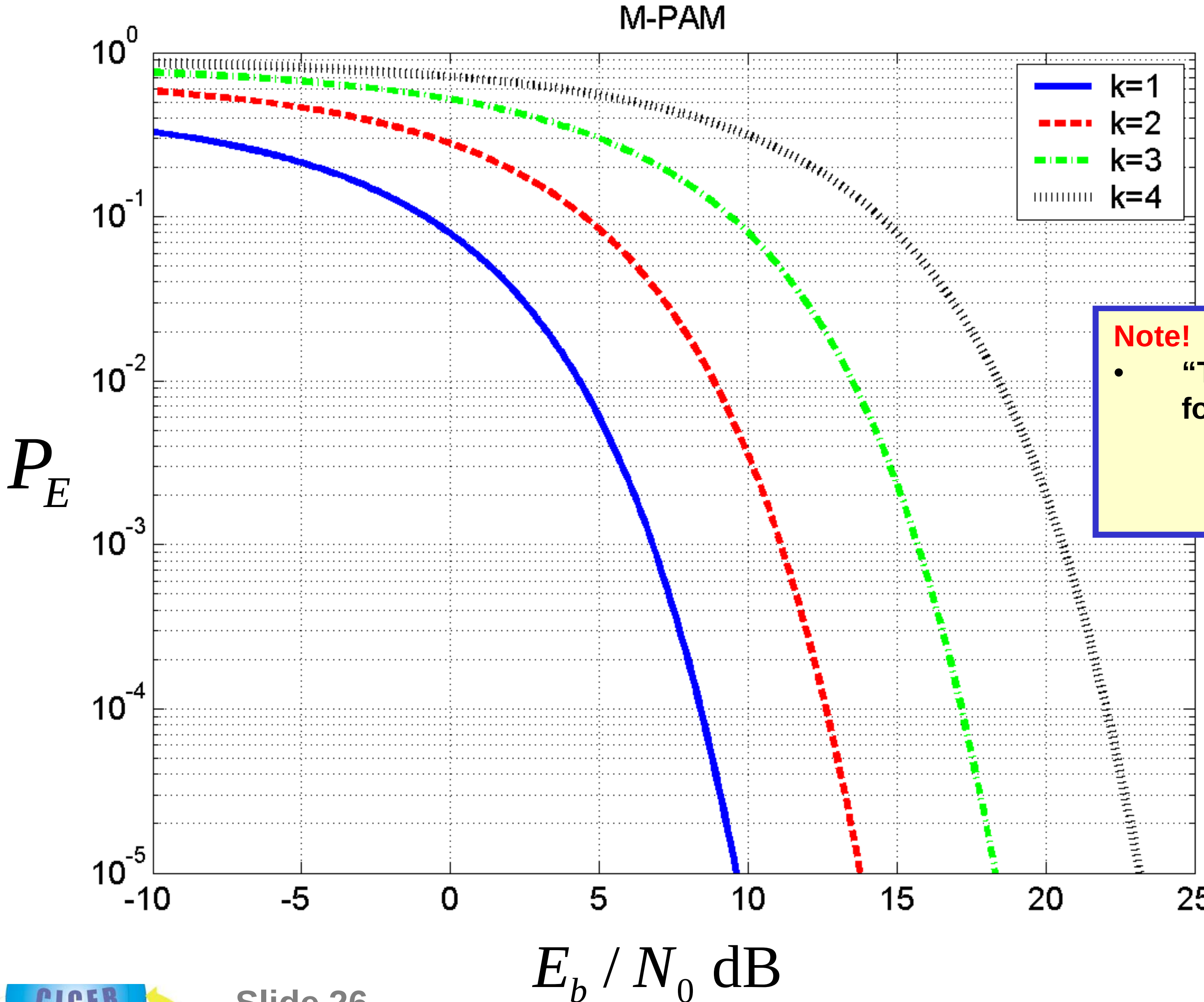
Probability of symbol error for M-FSK



Note!

- **"The same average symbol energy for different sizes of signal space"**

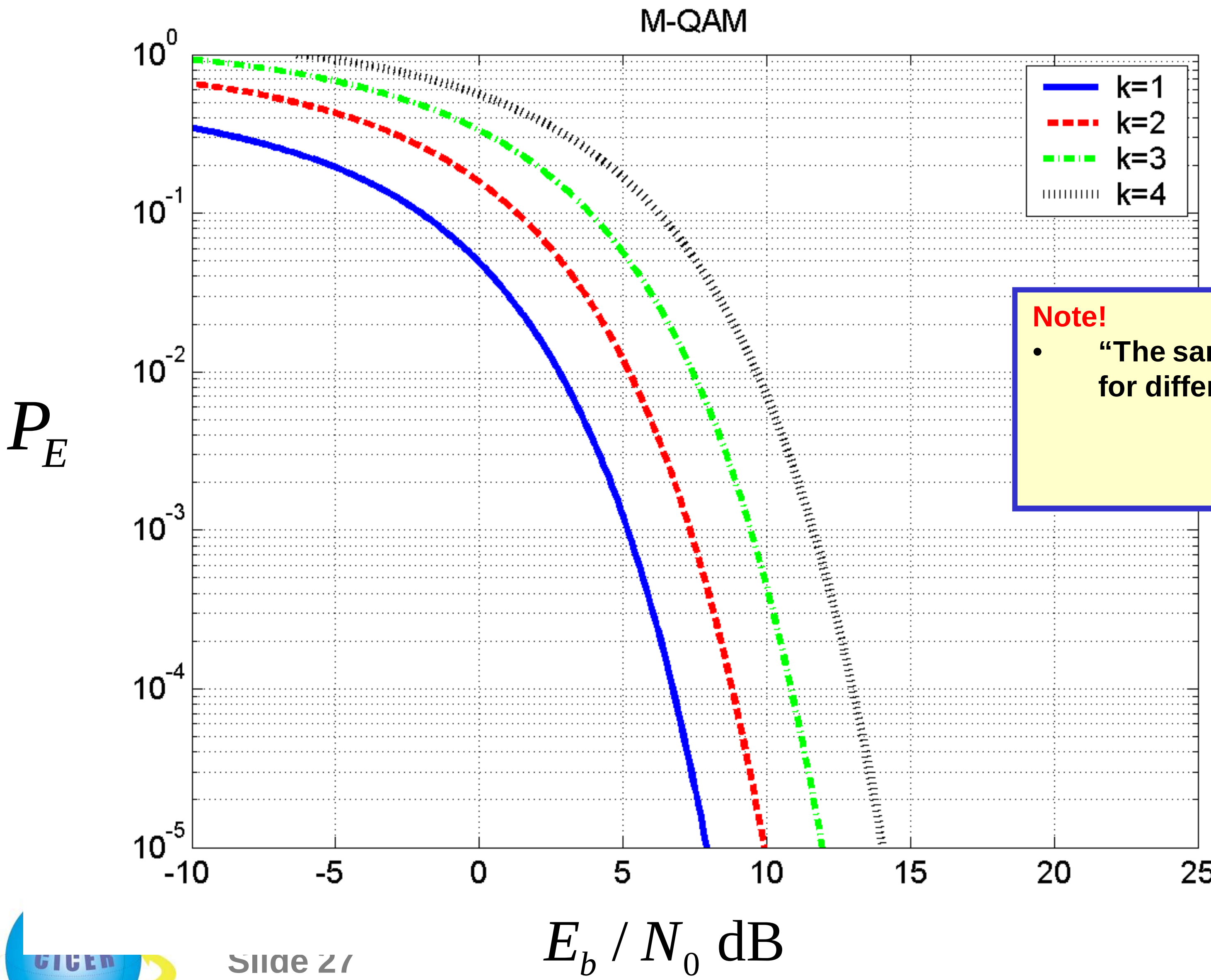
Probability of symbol error for M-PAM



Note!

- **"The same average symbol energy for different sizes of signal space"**

Probability of symbol error for M-QAM



Note!

- **"The same average symbol energy for different sizes of signal space"**